CONTENTS

Preface		v
	1 THE NEUTRON TRANSPORT EQUATION	1
1.1 De	erivation of the Transport Equation	1
1.1a	Introduction	1
1.1b	Definitions and Notation	2
1.1c	Derivation of the Neutron Transport Equation	11
1.1d	Interface and Boundary Conditions	15
1.1e	Conservation Relations	17
1.1f	Linearity of the Transport Equation: Green's Function	19
1.2 In	legral Equation for Neutron Transport	21
1.2a	Introduction	21
1.2b	Derivation of the Integral Equation	22
1.2c	Isotropic Scattering and Source	25
1.2d	Anisotropic Scattering	27
1.3 Th	e Transport Equation for Special Geometries	28
1.3a	Plane and Spherical Geometries	28
1.3b	Conservation Form for Curved Geometries	30
1.3c	Special Forms of the Integral Equation	32
1.4 Li	mitations of the Neutron Transport Equation	35
1.4a	Introduction	35
1.4b	Neutron as a Point Particle	35
1.4c	The Expected (or Probable) Value	36
1.4d	Delayed Neutrons	37

vii

viii CONTENTS

1.5	General Properties of Solutions of the Time-Dependent Transport	
	Equation	37
1.5	a The Criticality Condition: General Considerations	37
1.5	b Spectrum of the Transport Operator and Criticality	39
1.5	c Results of Rigorous Analysis of the Criticality Condition	42
1.5	1	43
1.5	` , ,	44
1.5	f Comparison of k and α Eigenvalues	47
1.6	Introduction to Methods of Solving the Neutron Transport Equation	48
1.6	a Need for Approximations	48
1.6	b Variations of Cross Sections with Energy	48
1.6	c Anisotropy of Neutron Emission	49
1.6	d Multigroup Methods	51
1.6	e The Monte Carlo Method	53
1.7	Appendix	56
1.7	a General Coordinate Systems	56
Exerc	ises	59
Refer	ences	61
	2 ONE-SPEED TRANSPORT THEORY	64
2.1	The One-Speed Transport Equation	64
	a Introduction	64
	b Derivation of the One-Speed Transport Equation	65
	c Infinite Plane Geometry	66
	d Use of Green's Function	68
2.2	Solution of the One-Speed Transport Equation by the Separation	
	of Variables	69
2.2	a Introduction	69
	b Source-Free Infinite Medium: Asymptotic Solutions	69
	c Infinite Medium Continuum (Singular) Solutions	73
2.2	d Completeness and Orthogonality of the Elementary Solutions	74
2.2	e Infinite Medium with Plane Source	75
2.2	f Point and Distributed Sources	78
	Solution of the One-Speed Transport Equation by the Fourier	
,	Transform Method	79
2.3	a Introduction	79
2.3	b Infinite Medium Isotropic Source	79

CONTENTS	ix

2.3c 2.3d	Asymptotic and Transient Solutions Infinite Medium Anisotropic Plane Source	80 84
	olution of the One-Speed Transport Equation by the Spherical armonics Method	86
2.4a 2.4b 2.4c	Introduction Infinite Medium Plane Isotropic Source Diffusion Theory and Diffusion Length	86 86 89
2.5 Tł	ne One-Speed Transport Equation in a Finite Medium	91
2.5a 2.5b 2.5c 2.5d 2.5e	Introduction The Milne Problem The Critical Slab Problem Spherical Harmonics Method with Boundary Conditions Adjacent Half-Spaces	91 93 95 97 99
2.5f	Spherical Geometry	101 102
2.6 Ai 2.6a 2.6b 2.6c 2.6d	Plane Geometry: Spherical Harmonics Diffusion Theory and the Transport Cross Section The Asymptotic Relaxation Length General Solution by Separation of Variables	102 104 105 107
2.7 Re	eciprocity Relations	108
2.7a 2.7b	Derivation of the General Relation Applications of the Reciprocity Relation	108 110
2.8 Co	ollision Probabilities	115
	Introduction Escape Probabilities: The Chord Method The Dancoff Correction	115 115 122
Exercise	es	125
Referen	aces	126
3	NUMERICAL METHODS FOR ONE-SPEED PROBLEMS: SIMPLE $P_{\scriptscriptstyle N}$ APPROXIMATIONS	129
3.1 Ex	spansion of Flux in Legendre Polynomials for Plane Geometry	129
3.1a 3.1b 3.1c	Introduction Plane Geometry: Spherical Harmonics Expansion The P_N Approximation	129 130 132

x CONTENTS

3.1d	The P_1 Approximation	122
3.1d	Boundary and Interface Conditions	132
3.16	Boundary and Interface Conditions	134
3.2 Di	fference Equations in Plane Geometry	136
3.2a	Difference Equations in the P_1 Approximation	136
3.2b	Approximation Errors in the Difference Equations	138
3.2c	Solving the P_1 Difference Equations	139
3.2d	Difference Equations in Diffusion Theory	142
3.2e	Solution of the P_N Equations	143
3.3 Fl	ux Expansion in Spherical and General Geometries	144
3.3a	Expansions in Spherical Geometry	144
3.3b	Boundary Conditions in Spherical Geometry	145
3.3c	Difference Equations in Spherical Geometry	146
3.3d	Expansions in General Geometry	146
3.3e	The P_1 Approximation in General Geometry	147
3.3f	The P_1 Approximation in One-Dimensional Geometries	150
3.4 Th	e Diffusion Equation in Two Dimensions	151
3.4a	Difference Equations in Two Dimensions	151
3.4a 3.4b		153
	Two-Dimensional Difference Equations in Matrix Form	154
3.4c	Solving the Matrix Equations by Iteration	
3.4d	Improved Iteration Procedures	156
3.4e	Difference Equations for More General Cases	158
3.5 Th	e Double- P_N Approximation	158
3.5a	Discontinuity of Angular Flux at an Interface	158
	Yvon's Method	161
3.6 Re	actor Cell Calculations	163
3.6a	The Wigner-Seitz Approximation	163
3.6b	The Spherical Harmonics Method for Cylindrical Cells	165
3.6c	Use of Cell Calculations	167
3.00	Ose of Cell Calculations	107
3.7 Co	nclusion	168
3.7a	Other Methods for Solving the Transport Equation	168
3.8 Ap	pendix	169
Exercise	s	170
Referen	200	171
INCICICIO	A.3	1/1

CONTENTS xi

	4 SOLUTION OF THE TRANSPORT EQUATION BY MULTIGROUP METHODS	173
4.1 In	troduction	173
4.1a	Outline of the Multigroup Method	173
4.1b	Comments on Other Methods of Solution	173
4.1c	Treatment of Variables	174
4.2 Sp	pherical Harmonics Equations in Plane Geometry	175
4.2a	Introduction	175
4.2b	Expansion of the Scattering Function	175
4.2c	The Spherical Harmonics Equations	177
4.2d	The P_1 Approximation and Diffusion Theory	178
4.3 Th	ne P_N Multigroup Equations	181
4.3a	Energy Groups and Group Constants	181
4.3b	The P_1 Multigroup Equations	183
4.3c	A Simple Source Problem	185
4.4 Ei	genvalue Problems in Multigroup Theory	186
4.4a	The Reactivity Eigenvalue	186
4.4b	The Multiplication Rate Eigenvalue	187
4.4c	Eigenvalues and Eigenfunctions for Multigroup Diffusion Theory	188
4.4d	Solving the Eigenvalue Problem	190
4.4e 4.4f	Difference Equations for the Multigroup Eigenvalue Problem Analysis of the Multigroup Eigenvalue Problem in Diffusion	193
	Theory: Outer Iterations	194
4.4g	Outer Iterations in the Multigroup P_1 Approximation	197
4.4h	General Comments on the Eigenvalue Problem	198
4.5 De	etermination of Multigroup Cross Sections	199
4.5a	Microscopic Cross Sections	199
4.5b	Estimation of Within-Group Fluxes	200
4.5c	The B_N Method	201
4.5d	Overlapping Energy Groups	203
4.6 Oı	utline of a Multigroup Calculation	204
4.6a	Reactor Codes	204
4.6b	Computation of an Eigenvalue Problem	205
_	ppendix: Relationship Between P_1 , Age-Diffusion, and Other	
Th	neories	207
4.7a	The Lethargy Variable	207
4.7b	Elastic Scattering in Terms of Lethargy	207

xii CONTENTS

	Age-Diffusion Theory	208 209
4.7e	Multigroup Age-Diffusion Theory	211
Exerci	ses	212
Refere	ences	212
5	DISCRETE ORDINATES AND DISCRETE $S_{\scriptscriptstyle N}$ METHODS	214
5.1 I	ntroduction	214
5.1a 5.1b	1	214 215
5.2 I	Discrete Ordinates for One Speed in Plane Geometry	216
5.2c 5.2d 5.2e 5.2f	Discrete Ordinates and Spherical Harmonics Gauss Quadrature Parameters The Double- P_N Method in Discrete Ordinates Anisotropic Scattering	216 218 219 220 221 222 225
5.3 E	Discrete Ordinates for One Speed in Curved Geometries	226
5.3a 5.3b 5.3c 5.3d 5.3e	The Conservation Principle Derivation of the Difference Equations Solution of the Difference Equation	226 228 229 232 236
5.4 N	fultigroup (Energy-Dependent) Problems	237
5.4a 5.4b 5.4c 5.4d	Determination of Group Constants Multigroup Discrete Ordinates Calculations	237 239 242 243
Exercis	ses	249
Refere	nces	249
6	THE ADJOINT EQUATION, PERTURBATION THEORY, AND VARIATIONAL METHODS	252
6.1 T	he Adjoint Function and its Applications	252
6.1a 6.1b		252 254

CONTENTS	xiii

6.1c	The Adjoint to the Transport Operator	254
6.1d	The Adjoint Function and Neutron Importance	256
6.1e	Adjoint of Green's Functions	258
6.1f	The One-Speed Adjoint Equation	259
6.1g	One-Speed Reciprocity Relation	261
6.1h	The Adjoint Integral Transport Equation	261
6.1i	Direct Derivation of an Equation for the Neutron Importance	262
6.1j	Spectrum of the Adjoint Operator and Criticality	264
6.1k	Interpretations of the Time-Dependent Adjoint Function	266
6.1m	Expansion of Time-Dependent Solutions	268
6.2 Tł	ne Adjoint Operators in Approximate Methods	269
6.2a	Introduction	269
6.2b	One-Speed P_1 , Diffusion, and S_N Theories	270
6.2c	Multigroup P_1 and Diffusion Theories	272
6.3 Pe	erturbation Theory	273
6.3a	Applications of Perturbation Theory	273
6.3b	Perturbation of the Multiplication Rate Constant, α	274
6.3c	Perturbation of the Effective Multiplication Factor	277
6.3d	Perturbation of a Critical System	279
6.3e	Perturbations in Multigroup Diffusion Theory	281
6.3f	An Application of Perturbation Theory	283
6.4 Va	ariational Methods	290
6.4a	Applications of Variational Methods	290
6.4b	Evaluation of Flux-Weighted Integrals	291
6.4c	Determination of Eigenvalues	295
6.4d	Applications of Variational Methods to One-Speed Problems	295
6.4e	An Absorption Probability Problem	298
6.4f	Discontinuous Trial Functions	301
6.4g	The J Functional as a Lagrangian	303
6.4h	Variational Derivation of Multigroup Equations	305
6.4i	Self-Consistent Determination of Group Constants	308
6.4j	Other Applications of Variational Methods	310
Exercise	es	312
Referen	nces	313
	7 NEUTRON THERMALIZATION	315
7.1 G	eneral Considerations	315
7.1a	Introduction	315
7.1b	Thermal Motion of Scattering Nuclei	317
	Alexand of Seminary (Mele)	511

xiv CONTENTS

	Chemical Binding Interference Effects: Coherent and Incoherent Scattering	317 320
7.2 Ge	eneral Features of Neutron Thermalization	323
7.2a	The Maxwell Distribution	323
7.2b	The Transport Equation for Thermal Neutrons	325
7.2c	Reciprocity Relation for Thermal Neutrons	327
7.3 No	eutron Scattering Laws	329
7.3a	Scattering from a Monatomic Gas	329
7.3b	The Scattering Function for a Monatomic Gas	331
7.3c	The Energy Transfer Function for a Monatomic Gas	333
7.3d	The General Scattering Law	337
7.3e	The Incoherent Approximation	340
7.4 Sc	attering in Bound-Atom Systems	341
7.4a	Results of Quantum Mechanical Calculations	341
7.4b	Intermediate Scattering Function for Monatomic Gas	342
7.4c	Isotropic Harmonic Oscillator	343
7.4d	Scattering by Real Crystalline Solids: Cubic Crystals	347
7.4e	Liquids: Model of the Diffusing Atom	350
7.4f	The Gaussian Approximation	350
7.4g	Experimental Determination of Scattering Laws	353
7.4h	Applications to Actual Moderators	354
7.5 Th	ermalization and Neutron Transport	362
7.5a	Introduction	362
7.5b	The Method of Collision Probabilities	364
7.6 Eig	genvalues and Thermalization Problems	366
7.6a	Introduction	366
7.6b	Types of Eigenvalue Problems	367
7.6c	Existence of the Eigenvalues	369
7.6d	Calculation of Eigenvalues and Eigenfunctions	373
7.6e	Eigenvalues in Diffusion Theory	375
7.6f	Deviations from the Maxwell Distribution	378
7.7 Ap	ppendix	383
7.7a	Source of Thermal Neutrons from Slowing Down	383
Exercise	s	384
References		385

CONTENTS xv

	8 RESONANCE ABSORPTION	389
8.1 R	esonance Cross Sections	389
8.1a	Introduction	389
8.1b		39
8.1c	Experimental Determination of Resonance Parameters	398
8.1d	Doppler Broadening	40
8.1e	Overlap and Interference of Resonances	400
8.1f	Resonance Absorption at Low Energies	409
8.2 T	he Unresolved Resonance Parameters	410
8.2a	Introduction	410
8.2b	Decay Channels and Level Width Distribution	411
8.2c	Resonance Peak (or Level) Spacings	41:
8.2d	Average Resonance Parameters	417
8.3 R	esonance Absorption in Homogeneous Systems	420
8.3a	Effective Resonance Integral	420
8.3b	Evaluation of Neutron Flux	422
8.3c	The Narrow Resonance Approximation	423
8.3d	Absorption Probability in the NR Approximation	427
8.3e	Doppler Broadening in the NR Approximation	431
8.3f	The NRIM Approximation	434
8.3g	Improved and Intermediate Approximations	436
8.3h	Resonances and Multigroup Constants	438
8.3i	Strongly Overlapping Resonances	439
8.4 R	esonance Absorption in Heterogeneous Systems	443
8.4a	Method of Collision Probabilities	443
8.4b	Equivalence Relations	446
8.4c	Numerical Computation of Resonance Integrals	450
8.4d	Approximate Dependence on Geometry	451
8.4e	The Doppler Effect in Fast Reactors	453
8.5 C	omparison of Theory and Experiment	454
8.5a	Thermal Reactors	454
8.5b	Fast Reactors	457
Exercis	es	458
Referer	nces	459

xvi CONTENTS

9 REACTOR DYNAMICS: THE POINT REAC RELATED MODELS	TOR AND 463
9.1 Introduction	463
9.1a Time-Dependent Problems	463
9.1b The Transport Equation with Delayed Neutrons	464
9.1c Feedback Effects	467
9.2 The Point Reactor	468
9.2a The Amplitude and Shape Factors	468
9.2b The Reactor Kinetics Equations	470
9.2c The Shape Factor	472
9.2d The Zero-Power Point Reactor	476
9.2e Asymptotic Period-Reactivity Relation	477
9.2f Numerical Solutions of the Point-Reactor Equation a	
Prompt-Lifetime Approximation	480
9.2g The Linearized Kinetics Equations	482
9.3 Transfer Functions	483
9.3a The Zero-Power Transfer Function	483
9.3b Sinusoidal Reactivity Perturbations	485
9.3c Space Dependence of Transfer Functions	488
9.4 The Point Reactor with Feedback	490
9.4a Introduction	490
9.4b The Transfer Function with Feedback	491
9.4c Stability Conditions	494
9.4d Power Limits for Stability	496
9.4e Stability and Reactivity Perturbation Frequency	499
9.4f Simple Models of Feedback	502
9.4g Other Sources of Instability	505
9.4h Relative Importance of Delayed and Prompt Neutro	
9.4i Feedback in a Nonlinear Point Reactor	508
9.5 Determination and Use of Transfer Functions	509
9.5a Introduction	509
9.5b The Reactor Oscillator Method	510
9.5c Correlation Methods	511
9.5d The Reactor Noise Method	513
9.5e Applications of the Transfer Function	514
9.6 Large Power Excursions	517
9.6a The Fuchs-Hansen Model	517

	CONTENTS	xvii
9.6b	Pulsed Fast Reactor	520
9.6c	Analysis of Fast-Reactor Accident	522
Append	lix	527
Exercises		528
Referen	ces	529
1	O SPACE-DEPENDENT REACTOR DYNAMICS AND RELATED TOPICS	532
10.1 S	pace and Time Dependent Neutron Transport Problems	532
10.1a	Methods of Solution	532
10.1b	Mode Synthesis and Expansion Methods	534
10.1c	, c	536
10.1d		542
10.1e		546
10.1f	*	554
10.1g	Xenon-Induced Power Oscillations	555
10.2 E	urnup Problems	562
10.2a	Introduction	562
10.2b	The Burnup Equations	564
10.2c	1 1	565
10.2d	*	568
10.2e	£ \	572
10.2f		573
10.2g	Flux Flattening with Burnable Poisons	576
10.3 C	alculations on Graphite-Moderated, Gas-Cooled Reactors	578
10.3a	Introduction	578
10.3b	Outline of the Calculational Methods	581
10.3c	Results of Cell Calculations	582
10.3d	1	586
10.3e	The Reactivity Temperature Coefficients	587
10.3f	Results for the Calder Hall Reactor	589
_	Results for the Peach Bottom Reactor	593
Exercises		598
Referen	ces	599
	APPENDIX: SOME MATHEMATICAL FUNCTIONS	603
The Delta Function		603
The Gamma Function		604

xviii	CONTENTS

The Error Function	604
The Exponential Integrals, $E_n(x)$	605
The Legendre Polynomials	606
The Associated Legendre Function	607
The Spherical Harmonics	608
References	610
Index	611