

Fig. 2. Average cosine of the scattering angle θ for H_2O as a function of incident neutron energy E . The open circles are at 200 C, all other points at room temperature.

A similar comparison for the organic moderators is made in Fig. 3.

Another important parameter in thermalization physics is M_2 , the second moment of energy transfer for a Maxwellian velocity distribution of the neutrons:

$$M_2 = \frac{1}{(k_B T)^4} \int_0^\infty dE \int_0^\infty dE' E e^{-\frac{E}{k_B T}} (E - E')^2 \sigma(E \rightarrow E'). \quad (3)$$

With the harmonic approximation for the scattering law one can write¹²

$$M_2 = \frac{15 \sigma_b}{16 A} \int_{-\infty}^{+\infty} \frac{t \dot{w}(t) \cdot dt}{\left\{ 0.25 + t^2 + \frac{w(t)}{A} \right\}^{7/2}} \quad (4)$$

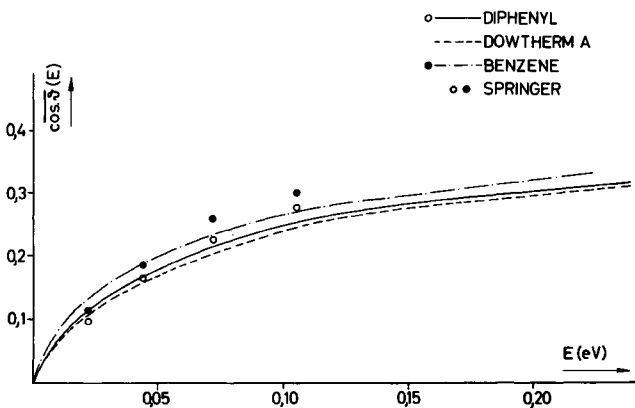


Fig. 3. Average cosine as a function of E for benzene, diphenyl and Dowtherm A.

¹⁰CH. REINSCH and T. SPRINGER, *Z.f. Naturforschung* 16a, 112 (1961).

¹¹T. SPRINGER, unpublished, personal communication which is acknowledged.

¹²P. SCHOFIELD, BNL 770 (1962).

with

$$\dot{w}(t) = 2 \int_0^\infty \frac{p(\beta)}{\beta} \sin \beta t d\beta,$$

$$w(t) = 2 \int_0^\infty \frac{p(\beta)}{\beta^2} \left\{ \cosh \frac{\beta}{2} - \cos \beta t \right\} d\beta. \quad (5)$$

Results from numerical calculations of Eq. 4 are given in Table I.

TABLE I
Calculated M_2 in Barns

Substance	H_2O	benzene	diphenyl	Dowtherm A
M_2	46.5	52	42	41

For an ideal gas of mass one, M_2 is 57.6 barns. The value for H_2O agrees well with M_2 following from the Nelkin model ($M_2 = 47.9$ barns). But all the M_2 values given in the table are of the order of 30% higher than the results from pulsed neutron work.

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The Nordheim-Fuchs Excursion Model with Non-Linear Reactivity Feedback

In his recent letter to the editor, Scalettar¹ has shown that serious error can result if one uses an 'average' value for the heat capacity in the Fuchs-Nordheim reactor excursion model² when, in fact, the heat capacity varies with temperature. In the following note, two simple lemmas are proved which permit easy insight into this point and in addition, generalize to include cases of both a temperature-dependent heat capacity and a temperature-dependent temperature coefficient.

The reactor equations are

$$\frac{dP}{dt} = \frac{\delta k_p - f(T)}{\ell} P \quad (1)$$

and
$$C(T) \frac{dT}{dt} = P. \quad (2)$$

¹R. SCALETTAR, "The Fuchs-Nordheim Model with Variable Heat Capacity," *Nucl. Sci. Eng.* 16, p. 459, (1963).

²G. GOERTZEL, *Reactor Handbook, Physics*, Vol. 1, Chapter 1.6, AEC-3645, (1955).

Here $P(t)$ is the reactor power at time t , δk_p is the reactivity inserted above prompt criticality, ℓ is the prompt-neutron lifetime, $C(T)$ is the heat capacity and $-f(T)$ is the reactivity feedback due to the temperature rise to T .

Division of (1) by (2) followed by integration leads to

$$P - P_0 = \frac{\delta k_p \int C dT - \int f(T) C dT}{\ell} \quad (3)$$

We define the 'termination' of the excursion as the point where the power returns to the initial power P_0 . At this point we have

$$\delta k_p = \frac{\int f(T) C dT}{\int C dT} = \frac{\int g(E) dE}{\int dE} \quad (4)$$

where $g(E) = f(T(E))$.

From (4) we see that some simplification results if we consider the energy feedback $g(E)$ rather than the temperature feedback $f(T)$.

Equation (4) is applicable to the case of a prompt excursion in the presence of a negative feedback mechanism. It neglects the effects of delayed neutrons and heat transfer. Under these conditions (4) can be expressed in terms of the following lemma: "The initiating prompt-reactivity insertion is equal to the reactivity feedback averaged over energy from the start to the 'termination' of the excursion." The lemma permits a simple graphical solution to the excursion problem which we use below in order to make some qualitative points of interest.

First consider Fig. 1 which takes $g(E)$ as being proportional to energy. The simplest situation corresponding to this case is that of constant heat capacity and a constant temperature coefficient ($f(T) = aT$, where a is constant). Our lemma says that the energy release will be such that the two shaded areas in Fig. 1 are equal at the termination

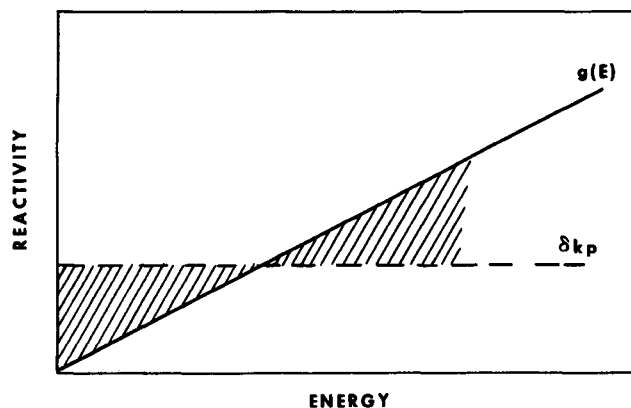


Fig. 1.

of the excursion. From the figure, one can immediately infer the well known result that the reactivity feedback at the end of the excursion is equal to twice δk_p . Furthermore, it is plain that this result holds regardless of the slope of $g(E)$, i.e., the magnitude of the temperature coefficient.

Now consider, for example, the case where the slope of $g(E)$ decreases with energy. This case corresponds to the situation discussed by Scalettar where $C(T)$ increases with temperature. It also corresponds to the case of a Doppler temperature coefficient decreasing with temperature.

Reference to Fig. 2 shows that under these conditions the reactivity feedback at the termination of the excursion will be less, but the energy release more than would be predicted if the slope of $g(E)$ were assumed to remain constant at its initial value. Even if one used the average slope to the point where $g(E) = \delta k_p$, the predicted energy release would be underestimated. Furthermore, if there is a constant temperature coefficient but a temperature-dependent heat capacity, Fig. 2 indicates that the temperature will be lower at the termination of the excursion than would be the case if the slope of $g(E)$ were assumed to remain constant (at any value). This follows because the reactivity feedback is directly proportional to temperature for constant temperature coefficient. It is interesting to note that these results which Scalettar obtained quantitatively for the case where $C(T) = C_0 + \gamma T$, γ a constant, depend only on the general shape of the curve of $g(E)$.

If Fig. 2 is taken to represent the case of constant heat capacity with a temperature-dependent temperature coefficient, then one can deduce that the temperature at the termination of the excursion will be higher than for the constant slope $g(E)$ whether its initial or its average value to the point where $g(E) = \delta k_p$ be used. This follows because under these conditions the temperature is directly proportional to the energy

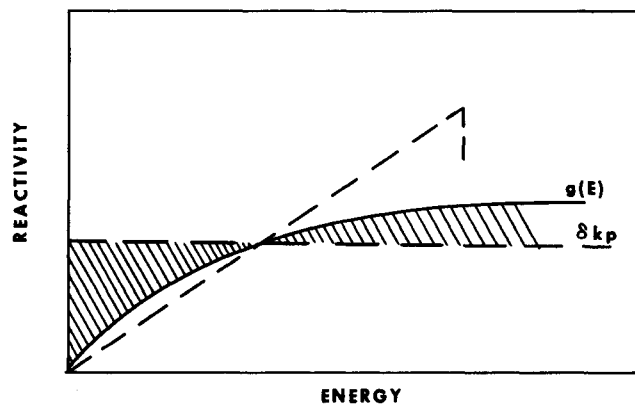


Fig. 2.

rather than the reactivity feedback. Thus, for the case where both the heat capacity and the temperature coefficient are temperature dependent, the temperature at the end of the excursion can be either greater or less than for the constant slope case, depending upon the details of the temperature-dependent quantities.

Another point which can be inferred from Fig. 2 is that when the slope of $g(E)$ is not constant, multiplication of $g(E)$ by a constant will change not only the energy release but also the reactivity feedback at the termination of the excursion. Furthermore, for a given $g(E)$, the ratio of the reactivity feedback to δk_p at the termination of the excursion depends upon δk_p itself. This contrasts with the case of constant slope discussed above.

Consider now the peak power reached during the excursion. Equation (3) can be rewritten in the form $1(P - P_0) = \delta k_p \int dE - \int g(E) dE$ or $P - P_0 =$

$\int \frac{(\delta k_p - g(E))}{\ell} dE$. This result can be stated in the form of a second lemma: "The increase of reactor power from the start of the excursion is proportional to the energy integral of the prompt reactivity. The constant of proportionality is the inverse of the prompt-neutron lifetime." The validity of this lemma rests on the same assumptions as for the first lemma.

The peak power will be reached when $g(E) = \delta k_p$. Thus, we see from the second lemma that in Fig. 2 the peak power will be proportional to the shaded area to the left of the intersection of the $g(E)$ curve and the δk_p line. From Fig. 2 it can be seen that peak power is greater than would be the case if the $g(E)$ curve retained its initial slope. On the other hand the peak power is less than would be predicted if one used the average slope of $g(E)$ up to the point where $g(E) = \delta k_p$.

In the constant slope case, one can immediately conclude from Fig. 1 that the peak power is proportional to the square of the initial reactivity insertion δk_p . Also, one can deduce that the peak power is inversely proportional to the temperature coefficient. No such simple relationships exist for the general case of non-constant slope.

It is sometimes helpful to extend simple theories by defining appropriate 'average' or 'effective' values for important parameters. However, an important conclusion from the figures and discussion above is that it is not generally possible to choose an average (constant) slope for $g(E)$ which will correctly give both the energy release at the 'termination' of the excursion and peak power during the excursion. (By choosing a constant slope one, of course, immediately precludes the possibility of correctly obtaining the reactivity feedback at the 'termination' of the

excursion.) Thus, the simple Nordheim-Fuchs' model should be approached with caution if either the heat capacity or the temperature coefficient varies significantly with temperature. One should use similar caution with more elaborate models which include heat transfer and delayed-neutron effects but ignore the temperature dependence of the important parameters.

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A Simple Method of Fitting SOFOCATE Thermal Constants for Highly Enriched Water Moderated Reactors

In some applications, such as core depletion studies, detailed thermal spectrum calculations cannot be performed for each case, owing to the large number of cases involved. Consequently a very simple spectrum model such as the Maxwellian has usually been used, which leads to errors that are not entirely mutually compensating. The SOFOCATE¹ model is a more accurate representation of the thermal spectrum, and Calame *et al.*², have described a variational technique for obtaining fitted SOFOCATE constants for use in depletion studies. This note describes an alternative and simpler method of comparable accuracy, which enables fitted SOFOCATE constants to be obtained by hand calculation, or with very small expenditure of computer time.

The scheme is based on the fact that for hydrogen moderated mixtures with approximately $1/v$ absorption, the SOFOCATE spectrum, and hence the spectrum average of any cross-section, is a function of only two variables, the moderator temperature and the ratio of 2200 m/s macroscopic absorption cross-section to the hydrogen number density.

It is found that any cross-section without a pronounced resonance peak in the thermal group may

¹H. AMSTER and R. SUAREZ, "The Calculation of Thermal Constants Averaged over a Wigner-Wilkins Flux Spectrum: Description of the SOFOCATE Code." WAPD-TM-39 (1957).

²G. P. CALAME, *et al.*, "A Two-Mode Variational Procedure for Calculating Thermal Diffusion Theory Parameters," *Nucl. Sci. Eng.* 10, 31-39 (May 1961).