TABLE I

Total Neutron Cross Section *vs* Neutron Energy for Pa^{233} from 0.01 to 1.0 eV

E(eV)	$\sigma(b)$	E(eV)	$\sigma(b)$	E(eV)	$\sigma(b)$
0.0109	81.04	0.0889	32.10	0.4476	26.73
0.0120	81.64	0.0977	30.51	0.4682	27.57
0.0132	81.29	0.1071	30.44	0.5021	23.80
0.0145	74.65	0.1179	28.38	0.5529	24.25
0.0160	70.05	0.1300	27.80	0.5989	24.41
0.0176	66.54	0.1428	27.47	0.6289	20.36
0.0193	67.22	0.1569	26.25	0.6515	21.32
0.0213	64.49	0.1725	25.67	0.6755	22.48
0.0234	61.43	0.1895	24.19	0.7008	25.72
0.0257	56.48	0.2040	23.51	0.7275	26.33
0.0283	54.02	0.2145	23.79	0.7485	35.48
0.0311	51.18	0.2260	23.01	0.7630	48.16
0.0343	47.78	0.2371	24.01	0.7780	53.57
0.0377	45.25	0.2490	25.66	0.7934	53.30
0.0414	44.30	0.2619	24.61	0.8093	61.04
0.0455	40.39	0.2807	23.51	0.8257	42.64
0.0501	39.01	0.3091	25.37	0.8425	35.91
0.0552	37.13	0.3419	24.86	0.8689	29.65
0.0606	36.97	0.3675	29.90	0.9059	34.07
0.0666	34.96	0.3853	31.98	0.9453	36.24
0.0734	33.85	0.4046	35.33	0.9874	31.14
0.0807	32.71	0.4253	29.02	1.0207	35.97

time, and the error in the half-life of Pa^{233}, increases the error to ± 1.6 barns. There are also other errors which may exist which have not been taken into consideration, such as sample nonuniformity and possibly small unknown contaminants. The final result of this determination gives a value for the 2200 m/sec total cross section of Pa^{233} as 57^{+3}_{-4} barns.

As an independent check on the original amount of Pa^{233} present in the sample at the time of chemical separation, measurements were taken on the sample after about three half-lives of decay. At this time, approximately 90% of the Pa^{233} had decayed to U^{233}. Figure 2 shows the U^{233} total cross section data obtained from this measurement. A least squares fit to these data gives a thermal total neutron cross section value of 589 barns which is in agreement with other values[5,6,7].

It is of interest to compare the present results with previously obtained integral measurements of the Pa^{233} cross section. The present 2200 m/sec value of 57^{+3}_{-4} barns for the total cross section agrees favorably with the 41 ± 5 barns and 39 ± 5 barns for the absorption cross section recently obtained by Halperin *et al.*[8] and by Eastwood and Werner[9] by activation and mass-spectrographic techniques respectively. However, it is a little lower than the value of 68 ± 6 barns obtained by Smith *et al.*[10] for the absorption cross section. It might also be noted that the resonance absorption integral estimated from preliminary resonance parameters obtained in the present work agrees, within the quoted errors, with resonance absorption integrals measured by activation[9] and mass-spectrographic techniques[11].

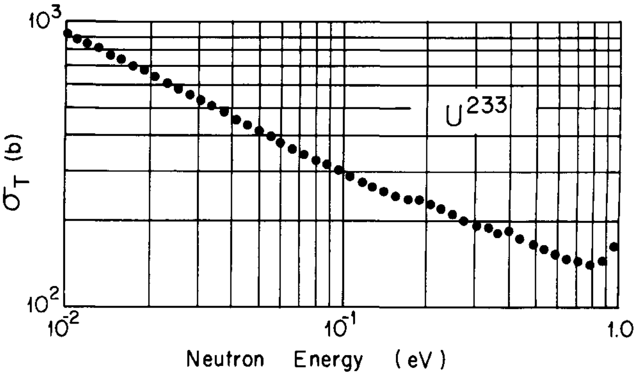

Fig. 2. The total neutron cross section as a function of neutron energy for U^{233} from 0.01 to 1.0 eV obtained from the Pa^{233} sample after about three half-lives of decay.

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[5] O. D. SIMPSON, M. S. MOORE and F. B. SIMPSON, *Nucl. Sci. Eng.* **7,** 187 (1960).

[6] R. C. BLOCK, G. G. SLAUGHTER and J. A. HARVEY, *Nucl. Sci. Eng.* **8,** 112 (1960).

[7] G. J. SAFFORD, W. W. HAVENS and B. M. RUSTAD, *Phys. Rev.* **118,** 799 (1960).

[8] J. HALPERIN *et al.*, private communication.

[9] T. A. EASTWOOD and R. D. WERNER, *Can. J. Phys.* 38, 751 (1960).

[10] R. R. SMITH *et al.*, IDO-16226 (1955).

[11] J. HALPERIN *et al.*, ORNL-3320, 1 (1962).

Inelastic Neutron Scattering by some Hydrogenous Moderators

In the Fermi pseudopotential approximation[1] the differential scattering cross section for a system of nuclei which is isotropic with respect to the direction of the incident neutron can be written:

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[1] P. A. EGELSTAFF, "Inelastic Scattering of Neutrons in Solids and Liquids," IAEA, Vienna 25 (1961).

$$\sigma(E\rightarrow E',\theta) = \frac{\sigma_b}{2k_BT}\sqrt{\frac{E'}{E}}\; e^{-\beta/2}\; S(\alpha,\beta), \qquad (1)$$

where E, E' are the energies of impinging and outgoing neutron,
θ is the scattering angle, and
α, β have the following meanings:

$$\alpha = \frac{1}{Ak_BT}(E' + E - 2\sqrt{E'E}\cos\theta), \quad \beta = \frac{E' - E}{k_BT}. \qquad (2)$$

Here A is the mass number of the principal scatterer,
k_BT is the temperature in energy units,
$S(\alpha,\beta)$ is the symmetrized form of the scattering law, which fulfills detailed balance.

A complete measurement of $S(\alpha,\beta)$ would yield all information necessary for calculations including scattering effects of the system. Using the rotating crystal time-of-flight spectrometer technique[2] scattering laws for benzene, diphenyl and Dowtherm A at room temperature have been measured. Primary energy resolution was 3% at 0.025 eV, time resolution 12 μsec/m and angular resolution 3°. With incident energies between 0.018 and 0.08 eV and scattering angles between 15° and 125° $S(\alpha,\beta)$-values for energy transfers between 0 and 0.12 eV and Q-values between 0 and 12 Å^{-1} (where $\hbar Q$ is the momentum transfer) have been determined. For small energy transfers a correction for the resolution of the apparatus has been made by fitting normal distributions to the incident and scattered neutron distributions.

No marked differences between the experimental $S(\alpha,\beta)$-values for diphenyl and Dowtherm A and the room temperature terphenyl measurements of Brugger[3] have been found. However, the benzene results differ considerably from these values in the region of small energy transfers $(0 < \beta \leq 1.5)$.

The measurements do not meet the whole range of interest in energy and momentum transfer. Therefore some theoretical guidance for extrapolation and interpolation of the experimental results is necessary. We determined generalized frequency distributions $p(\beta)$ by extrapolating the measured S/α to $\alpha = 0$ where S is per H-atom. This method first described by Egelstaff[1] is useful if a harmonic approximation can be made and if one has mainly to do with the incoherent part of $S(\alpha,\beta)$ which is the case with the materials we have measured because of the large hydrogen cross section. Using a modified version of the computer programs LEAP and ADDELT[4a] we determined $p(\beta)$ in an iterative way starting with a first experimental guess. The high energy modes of the benzene ring have been taken into account in an approximative manner using 4 δ-functions at 4.5, 5.4, 7.1 and 15 k_BT. The $p(\beta)$ we got in this way are shown in Fig. 1; the peaks of the distributions are in good agreement with infrared and Raman measurements. The low-energy peak, which is most pronounced in benzene, is probably due to torsional vibrations and has also been observed in Raman measurements[5]. The peaks at higher energies correspond to vibrational modes of the molecules.

Starting from $p(\beta)$ with LEAP and KAPIXSE which is essentially a part of the Winfrith PIXSE[a] program[6], 55 by 55 scattering matrixes have been calculated for further use in the calculation of thermal spectra. In these calculations we also got $\overline{\cos\theta}(E)$, the total scattering cross section $\sigma_s(E)$, and $\lambda_{tr}(E)$, the transport mean free path. In Fig. 2 results for $\overline{\cos\theta}(E)$ of H_2O using a $p(\beta)$ measured by Haywood[7] are compared with Nelkin kernel calculations[8] and more direct measurements[9-11].

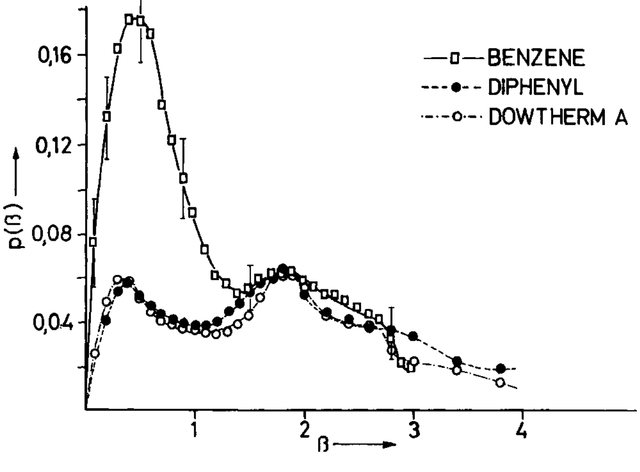

Fig. 1. Value of $p(\beta)$ from extrapolation of S/α to $\alpha = 0$. The indicated errors are due to uncertainties in the extrapolation process.

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[2] W. GLÄSER, "Inelastic Scattering of Neutrons in Solids and Liquids," Vol. I, IAEA, Vienna 307 (1963).

[3] R. M. BRUGGER, *Phys. Rev.* 126, 29 (1962).

[a] The authors are indebted to Drs. P. Egelstaff and P. Schofield for making the computing programs available.

[4] P. A. EGELSTAFF and P. SCHOFIELD, *Nucl. Sci. Eng.* 2, 260 (1962).

[5] A. FRUHLING, *Ann. de Phys.* 6, 401 (1951).

[6] J. D. MACDOUGALL, AEEW-M 318 (1963).

[7] B. C. HAYWOOD and I. M. THORSON, BNL 719, Vol. I, 26 (1962).

[8] E. KIEFHABER, *Nukleonik* 4, 82 (1962).

[9] W. L. WHITTEMORE and A. W. McREYNOLDS, "Inelastic Scattering of Neutrons in Solids and Liquids," IAEA, Vienna 511 (1961).

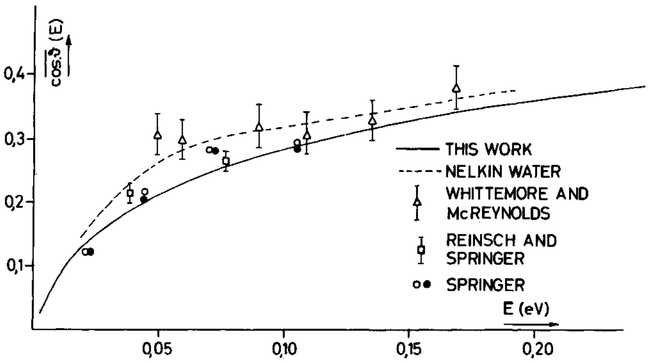

Fig. 2. Average cosine of the scattering angle θ for H_2O as a function of incident neutron energy E. The open circles are at 200 C, all other points at room temperature.

A similar comparison for the organic moderators is made in Fig. 3.

Another important parameter in thermalization physics is M_2, the second moment of energy transfer for a Maxwellian velocity distribution of the neutrons:

$$M_2=\frac{1}{(k_BT)^4}\int_0^\infty dE\int_0^\infty dE'Ee^{-\frac{E}{k_BT}}\,(E-E')^2\sigma(E\rightarrow E')\,. \tag{3}$$

With the harmonic approximation for the scattering law one can write[12]

$$M_2=\frac{15}{16}\frac{\sigma_b}{A}\int_{-\infty}^{+\infty}\frac{t\dot{w}(t)\cdot dt}{\left\{0.25+t^2+\frac{w(t)}{A}\right\}^{7/2}} \tag{4}$$

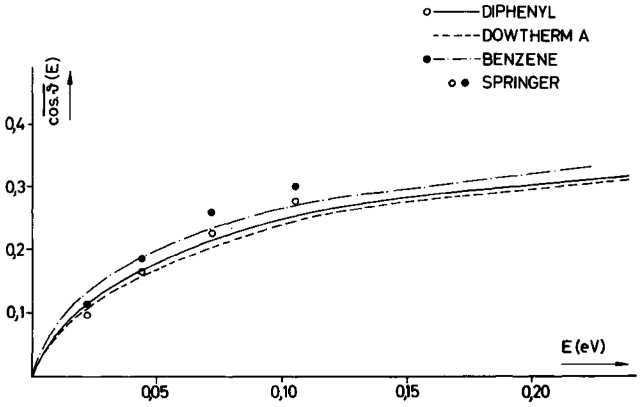

Fig. 3. Average cosine as a function of E for benzene. diphenyl and Dowtherm A.

with

$$\dot{w}(t)=2\int_0^\infty\frac{p(\beta)}{\beta}\sin\beta t d\beta,$$

$$w(t)=2\int_0^\infty\frac{p(\beta)}{\beta^2}\left\{\cos h\frac{\beta}{2}-\cos\beta t\right\}d\beta. \tag{5}$$

Results from numerical calculations of Eq. 4 are given in Table I.

TABLE I

Calculated M_2 in Barns

Substance	H_2O	benzene	diphenyl	Dowtherm A
M_2	46.5	52	42	41

For an ideal gas of mass one, M_2 is 57.6 barns. The value for H_2O agrees well with M_2 following from the Nelkin model (M_2 = 47.9 barns). But all the M_2 values given in the table are of the order of 30% higher than the results from pulsed neutron work.

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[10] CH. REINSCH and T. SPRINGER, *Z.f. Naturforschung* 16a, 112 (1961).

[11] T. SPRINGER, unpublished, personal communication which is acknowledged.

[12] P. SCHOFIELD, BNL 770 (1962).

The Nordheim-Fuchs Excursion Model with Non-Linear Reactivity Feedback

In his recent letter to the editor, Scalettar[1] has shown that serious error can result if one uses an 'average' value for the heat capacity in the Fuchs-Nordheim reactor excursion model[2] when, in fact, the heat capacity varies with temperature. In the following note, two simple lemmas are proved which permit easy insight into this point and in addition, generalize to include cases of both a temperature-dependent heat capacity and a temperature-dependent temperature coefficient.

The reactor equations are

$$\frac{dP}{dt}=\frac{\delta k_p-f(T)}{\ell}P \tag{1}$$

and

$$C(T)\frac{dT}{dt}=P\,. \tag{2}$$

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[1] R. SCALETTAR, "The Fuchs-Nordheim Model with Variable Heat Capacity," *Nucl. Sci. Eng.* 16, p. 459, (1963).

[2] G. GOERTZEL, *Reactor Handbook, Physics*, Vol. 1, Chapter 1.6, AECD-3645, (1955).