LETTERS TO THE EDITOR

BREEDING RATIO FOR FAST REACTORS

Dear Sir:

At the last American Nuclear Society Winter Meeting "An Improved Definition of the Breeding Ratio for Fast Reactors" was suggested by K. O. Ott.[1] The general definition of the internal breeding ratio of a region n is

$$BR_n = \frac{\sum_i \gamma_i C_{n,i-1}}{\sum_i \gamma_i A_{n,i}} \quad , \tag{1}$$

where i is a suffix that can take the values 5, 6, 8, 9, 0, 1, and 2 representing ^{235}U, ^{236}U, ^{238}U, ^{239}Pu, ^{240}Pu, ^{241}Pu, and ^{242}Pu, respectively; $C_{n,i}$, $A_{n,i}$, and $F_{n,i}$ [in Eq. (4)] denote, respectively, the capture, absorption, and fission rate for isotope i of region n; and the γ_i are weighting factors. The improved weighting factors suggested in Ref. 1 are

$$\gamma_i = \frac{\eta_i}{\eta_9} \quad \text{with} \quad \eta_i = \frac{\nu_i \sigma_{fi}}{\sigma_{ai}} \quad . \tag{2}$$

From these definitions it is possible to find the internal breeding gain of region n,

$$G_n = BR_n - 1 = \frac{\sum_i \gamma_i (C_{n,i-1} - A_{n,i})}{\sum_i \gamma_i A_{n,i}} \quad . \tag{3}$$

Since in a given region n the microscopic cross sections are assumed constant, one has

$$\sum_i \gamma_i A_{n,i} = \frac{1}{\eta_9} \sum_i \frac{\nu_i \sigma_{fi}}{\sigma_{ai}} A_{n,i} = \frac{1}{\eta_9} \sum_i \nu_i F_{n,i} \quad , \tag{4}$$

and thus, assuming that the ν_i are the same for all isotopes,

$$G_n = \frac{\sum_i g_i (C_{n,i-1} - A_{n,i})}{\sum_i F_{n,i}}$$

$$g_i = \frac{\sigma_{f,i}}{\sigma_{a,i}} \quad . \tag{5}$$

Now let us turn to the so-called British definition[2,3] of the internal breeding gain

$$G_n = \frac{\sum_i w_i (C_{n,i-1} - A_{n,i})}{\sum_i F_{n,i}}$$

$$w_i = \frac{(\nu \sigma_f - \sigma_a)_i - (\nu \sigma_f - \sigma_a)_8}{(\nu \sigma_f - \sigma_a)_9 - (\nu \sigma_f - \sigma_a)_8} \quad . \tag{6}$$

For a typical fast-reactor spectrum, one has

Isotope i	^{239}Pu	^{240}Pu	^{241}Pu	^{242}Pu
w_i	1.0	0.16	1.43	0.17
g_i	0.82	0.40	0.88	0.45
$\gamma_i \approx g_i/g_9$	1.0	0.49	1.07	0.55

Clearly the breeding ratio will be less sensitive to the plutonium isotopic composition with the weighting factors g_i than with w_i.

As stated by Ott, if the breeding gain was insensitive with respect to changes in the plutonium composition it is true that it would allow a good estimate of the equilibrium breeding gain and doubling time using only static reactor calculations. But unfortunately it is not insensitive, and there is no apparent physical ground to define the weighting factors as in Eq. (2).

As a matter of fact, the w_i are the relative reactivity worths of the different isotopes, and extensive fast-reactor physics calculations have shown that the critical mass for a given reactor is practically constant when expressed in equivalent ^{239}Pu.

The British definition of the breeding gain [Eq. (6)] is precisely the one that should be used to calculate the reactor doubling time. Indeed, the standard definition of the doubling time of a given reactor is the time needed to build up an amount of fuel material sufficient to make another similar reactor critical.

Such a definition of the doubling time cannot be readily derived from the definition of the breeding ratio given by Eq. (5).

More precisely, to start up a reactor one needs an initial fuel mass which is more than critical to compensate for the loss of reactivity during burnup. Since this loss of reactivity depends strongly on the fuel isotopic composition it is possible to calculate adequately the corresponding weighting factors w_i as was shown in Ref. 4. But there is then no simple definition of the weighting functions w_i as in Eq. (6), since they include not only the physical properties of an isotope but also those of all its daughter isotopes (produced during irradiation), and some economic factors too.

One might also define an "economic doubling time" as the time needed to build up an amount of heavy isotopes, the sale value of which is equal to the purchase cost of a more than critical mass sufficient to start up a similar reactor.

The weighting coefficients would then be

$$(w_i)_{\text{econ}} = \frac{P_i}{P_9} \quad ,$$

where P_i is the price for isotope i. Such prices P_i depend on the evolution of the world market and not on the kind of reactor we are considering. The practical interest of such a definition is, however, questionable as most of such prices are not fixed presently.

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REFERENCES

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3. A. R. BAKER, Letter to the Editor, *Nuclear News* (April 1967).

4. L. HERMANS and M. EGLÈME, "Valeur d'Équivalence des Divers Isotopes de l'Uranium et du Plutonium dans les Réacteurs à Neutrons Rapides," *IAEA - Symp. on Economics of Nuclear Fuel*, Gottwaldov (May 1968).

REPLY TO GOLDSCHMIDT

Dear Sir:

At the American Nuclear Society Meeting in San Francisco, 1969, I presented the paper[1] "An Improved Definition of the Breeding Ratio for Fast Reactors." The improvement consisted of a set of isotopic weight factors which reduce the sensitivity of the breeding gain to the composition of the fuel. The use of this improved definition avoids that an inaccuracy in the estimation of the equilibrium composition causes a large inaccuracy in the equilibrium breeding gain. The printed summary, limited to 450 words in addition to a figure, has been misinterpreted by Goldschmidt.[2] I present in this reply the more complete argumentation.

Goldschmidt states that the British definition of the breeding gain with weight factors w_i, representing the reactivity worths of isotope i, "is precisely the one that should be used to calculate the reactor doubling time." He states, in addition, that together with this definition one must "use the best average equilibrium composition." Goldschmidt's argumentation consists of the two parts which are the original basis of the British definition[3] of the breeding gain.

F1: "The doubling time of a given reactor is the time needed to build up an amount of fuel material sufficient to make another similar reactor critical."

F2: The isotope contribution to the critical mass is proportional to the reactivity worth of isotope i. This reactivity worth may be approximately calculated by average one group quantities.

The first point is obvious. The second point is long known. I used it, like others, in 1961 to calculate the contribution of the different plutonium isotopes to k_{eff}. On the basis of these two points Goldschmidt obviously concludes that "precisely the relative reactivity worths" should be used as weight factors in the calculation of the doubling time. In addition, Goldschmidt states by referring to his Eq. (7) that just the "fission rate" should be used as denominator in the breeding gain. In the first part of the following argumentation it is shown that one cannot conclude from the two facts, F1 and F2, which weight factors to use for the calculation of the equilibrium doubling time and also not which denominator to use in the definition of the breeding gain.

The first two questions I would like to discuss deal with the equilibrium fuel cycle which is the main subject of Goldschmidt's letter:

Q1: Is the use of the reactivity worths *necessary* for the calculation of the correct doubling time in an equilibrium state?

Q2: Is it *necessary* to use the fission rate as denominator of the breeding gain? [Comp. Eq. (7) in Ref. 2.]

DISCUSSION OF THE EQUILIBRIUM DOUBLING TIME (to Q1)

The equilibrium doubling time is commonly understood as the inverse "time constant" in an exponential accumulation of fuel. Depending on the average time Δt between the breeding events and the actual reuse of the bred fuel one defines the doubling time in two extreme cases:

t_{OD}, the doubling time without reuse

t_{CD} (= $\ln 2 \cdot t_{\text{OD}}$) the doubling time with instantaneous reuse, the "compound doubling time."

The actual doubling time (t_D) has a value between these two extremes depending on the time Δt. The questions to be discussed here are independent of this problem. We may therefore just use t_D without specifying its relation to t_{OD} and Δt.

Let $\hat{M}_f(t)$ be the accumulated "fuel" mass. The definition of "fuel" will be discussed below. $\hat{M}_f(t)$ is related to the doubling time by

$$\frac{\ln 2}{t_D} = \frac{1}{\hat{M}_f} \frac{d\hat{M}_f}{dt} \quad . \tag{1}$$

This leads to the exponential accumulation:

$$\hat{M}_f(t) = \hat{M}_f(o) \cdot \exp\left(\frac{\ln 2}{t_D} \cdot t\right) \quad . \tag{2}$$

For equilibrium operation the relative change of accumulated fuel is independent of the specific value of $M_f(t)$. Therefore, the right-hand side of Eq. (1) may be replaced by

$$\frac{1}{\hat{M}_f(t)} \cdot \frac{d\hat{M}_f(t)}{dt} = \frac{1}{M_f} \cdot \left(\frac{dM_f}{dt}\right) \quad , \tag{3}$$