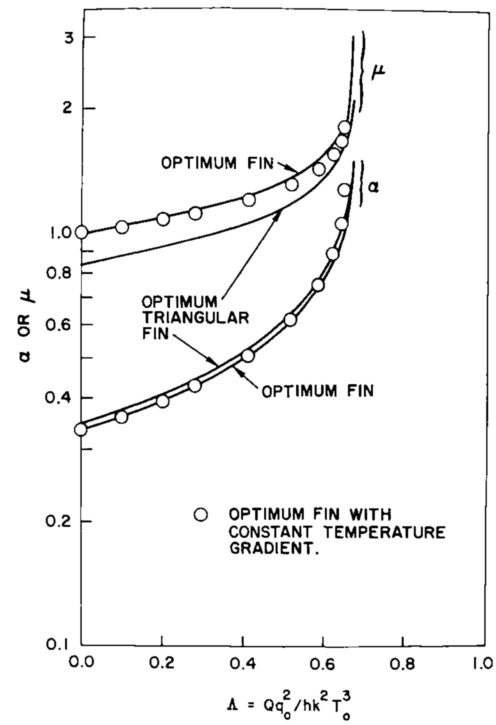

FIG. 2. Graphs of dimensionless minimum profile area $\alpha = h^2kT_0{}^3A_P/q_0{}^3$, and dimensionless fin height $\mu = whT_0/q_0$ for (i) optimum fin, (ii) optimum fin with constant temperature gradient, and (iii) optimum triangular fin.

and consequently we find that the function T defined in (6) will satisfy the boundary conditions (5) if and only if $C = 0$,

$$BI_1(\lambda) = (\lambda/4\mu) - (8\mu^2\Lambda/\lambda^3),$$

$$16\mu^3\Lambda[2\lambda I_0(\lambda) - (\lambda^2 + 4)I_1(\lambda)] + 4\mu\lambda^4 I_1(\lambda) = \lambda^5 I_0(\lambda). \quad (7)$$

Thus, the minimizing problem has been reduced to finding, for given Λ, a pair of values λ and μ satisfying (7) for which

$$\alpha = 4\mu^3/\lambda^2 \quad (8)$$

is a minimum. It follows from (8) and $d\alpha/d\lambda = 0$ that $d\mu/d\lambda = 2\mu/3\lambda$. If this value is substituted into the equation obtained by differentiating (7) with respect to λ, the result can be manipulated to read as follows:

$$16\mu^3\Lambda[(2\lambda - \lambda^3)I_0(\lambda) - (\lambda^2 + 4)I_1(\lambda)] + (4\mu\lambda^4/3)[11I_1(\lambda) + 3I_0(\lambda)] = \lambda^5[5I_0(\lambda) + \lambda I_1(\lambda)]. \quad (9)$$

Equations (7) and (9) are a pair of simultaneous linear equations in the variables μ and $\mu^3\Lambda$ and their solution is

$$\mu = \frac{3\lambda[(4\lambda + \lambda^3)I_1{}^2(\lambda) + 2(8 + \lambda^2)I_0(\lambda)I_1(\lambda) - (8\lambda + \lambda^3)I_0{}^2(\lambda)]}{8[4(4 + \lambda^2)I_1{}^2(\lambda) - 2\lambda I_0(\lambda)I_1(\lambda) - 3\lambda^2 I_0{}^2(\lambda)]}, \quad (10)$$

$$\Lambda = \frac{\lambda^5[3\lambda I_1{}^2(\lambda) + 4I_0(\lambda)I_1(\lambda) - 3\lambda I_0{}^2(\lambda)]}{32\mu^3[4(4 + \lambda^2)I_1{}^2(\lambda) - 2\lambda I_0(\lambda)I_1(\lambda) - 3\lambda^2 I_0{}^2(\lambda)]}. \quad (11)$$

Equations (8), (10), and (11) can now be used to calculate the values of α and μ plotted versus Λ on Fig. 2 for the triangular case. The maximum possible value of Λ is 0.6640, attained when $\lambda = 4.975$ and in this extreme case the dimensionless profile area α is 1.4746. It is seen that for small and moderate values of Λ the triangular fin is slightly inferior to the optimum fin, as well as to the optimum fin with constant temperature gradient, the inferiority decreasing with increasing Λ. For larger values of Λ the optimum triangular fin becomes superior to the optimum fin with constant temperature gradient and has an area almost indistinguishable from that of the optimum fin.

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J. ERNEST WILKINS, JR.

John Jay Hopkins Laboratory for Pure and Applied Science, General Atomic Division of General Dynamics Corporation, San Diego, California

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Xenon Spatial Oscillations*

In an earlier article (*1*), the threshold values of the flux for oscillations in the axial power distribution were estimated as a function of reactor size and degree of flux flattening, for a cylindrical reactor with a zero power coefficient. The purpose of this letter is to make available the results of more recent calculations that show the effect of nonzero power coefficients, and the threshold values for "flux tilt" oscillations in a cylindrical reactor with a flattened radial power distribution.

The mathematical development and the constants employed in the calculations are given in ref. *1*. In that paper it was shown that the flux threshold for oscillations and the corresponding oscillation periods could be obtained, with an error of less than 5%, from the amount of material buckling that must be added uniformly to the critical reactor to excite the first spatial harmonic orthogonal to the unperturbed power distribution. The relations among the threshold flux level for axial oscillations, the oscillation period, the additional buckling required to excite the first harmonic, $\mu_1{}^2$, and the power coefficient are given in Eqs. (10) and (11) of ref. *1*.

Figure 1 shows the effect of a nonzero temperature coefficient (*2*) on the flux threshold for xenon oscillations for an unflattened (cosine) power distribution in a slab reactor (or in the axial direction of a cylindrical reactor). The units of the temperature coefficient are milli-k of reactivity per unit power where unit power corresponds to a flux level of 6×10^{13} n/cm² sec. For other flux levels, if an effective temperature is defined as being directly proportional to the flux level, the temperature coefficient per unit effective temperature is assumed constant.

* The information contained in this article was developed during the course of work under contract AT(07-2)-1 with the U. S. Atomic Energy Commission.

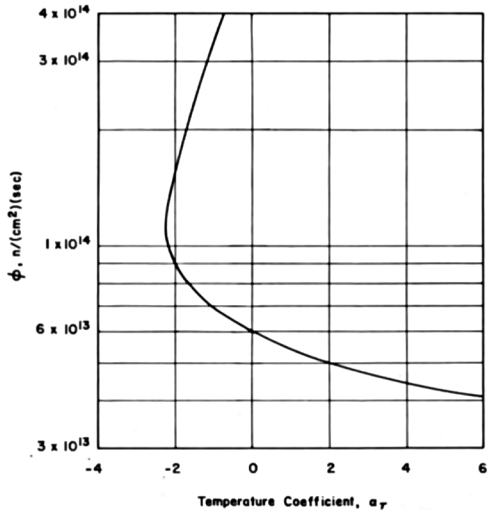

FIG. 1. Effect of temperature coefficients on xenon axial oscillation threshold. The units of the temperature coefficient are milli-k of reactivity per unit power where unit power corresponds to a flux level of 6×10^{13} n/cm² sec.

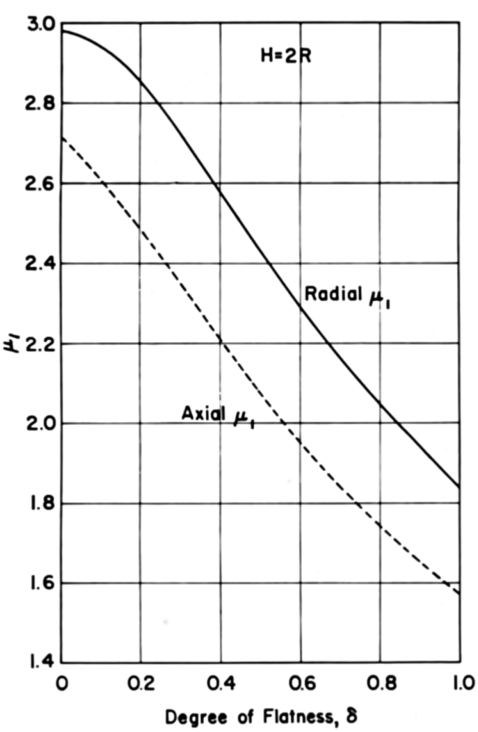

FIG. 2. Threshold parameter for reactor with flattened power distribution. Square root of buckling, required to excite the first harmonic instability for reactors with unit height and diameter.

For a cylindrical reactor with a power distribution that is flattened for a fraction δ of the total radius, the unperturbed power distribution, g_0, is given by

$$g_0 = \text{Constant} \qquad r \leqq \delta$$

$$= C_0 \left[Y_0(Br) - \frac{Y_0(B)}{J_0(B)} J_0(Br) \right] \qquad \delta \leqq r \leqq 1$$

The buckling, B^2, required in the outer region is a function of the degree of flattening, δ, as determined by

$$\frac{J_1(B\delta)}{J_0(B)} = \frac{Y_1(B\delta)}{Y_0(B)}$$

The first harmonic that is orthogonal to g_0 is

$$g_1 = A_1 J_1(\mu_1 r) \cos\theta \qquad r \leqq \delta$$

$$= C_1 \left[J_1(\alpha r) - \frac{J_1(\alpha)}{Y_1(\alpha)} Y_1(\alpha r) \right] \cos\theta \qquad \delta \leqq r \leqq 1$$

where

$$A_1 = \frac{C_1}{J_1(\mu_1 \delta)} \left[J_1(\alpha\delta) - \frac{J_1(\alpha)}{Y_1(\alpha)} Y_1(\alpha\delta) \right]$$

$$\alpha = \sqrt{B^2 + \mu_1^2}$$

The buckling that must be added to excite this first harmonic, μ_1^2, is a function of the flattening parameter, δ, as determined by the requirement for continuity of current, through the relation,

$$J_1(\alpha\delta) - \frac{J_1(\alpha)}{Y_1(\alpha)} Y_1(\alpha\delta) = \frac{\alpha J_1(\mu_1\delta)}{\mu_1 J_0(\mu_1\delta)} \left[J_0(\alpha\delta) - \frac{J_1(\alpha)}{Y_1(\alpha)} Y_0(\alpha\delta) \right]$$

Figure 2 shows this radial μ_1 as a function of δ. Figure 3 shows threshold values of the flux for radial power oscillations as a function of the reactor size R^2/M^2 and of the frac-

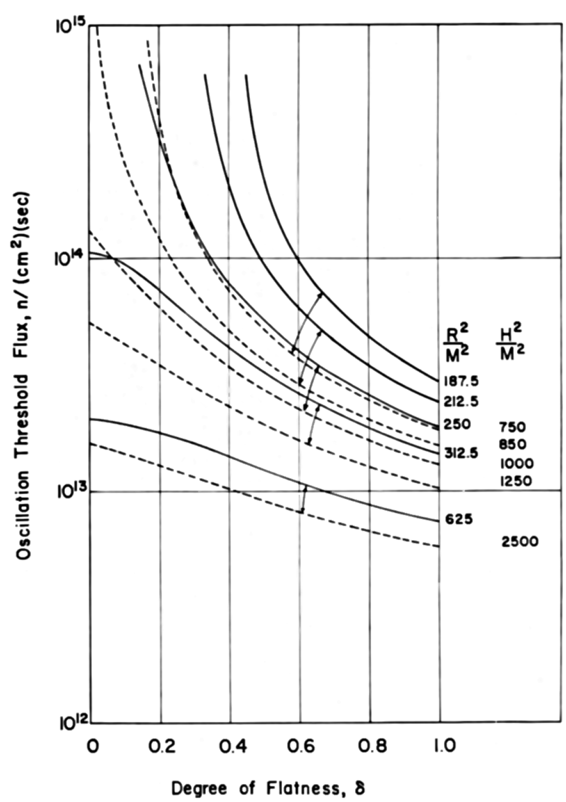

FIG. 3. Threshold flux values for spatial xenon instability in cylindrical reactors. ——— = flux for radial instability and -------------- = flux for axial instability. Values of R^2/M^2 are selected so that $R = 1/2H$ for corresponding curves.

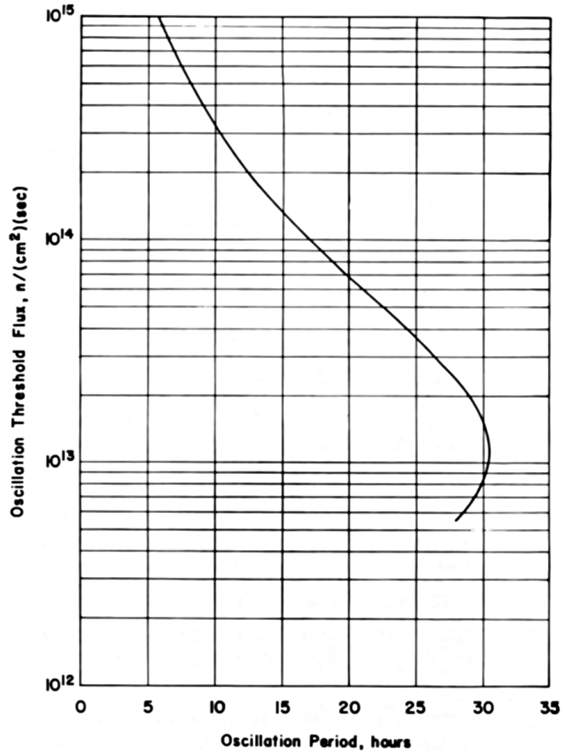

FIG. 4. Period of xenon oscillation as a function of the threshold value of the neutron flux.

tion of the radius that is flattened, for the case of a zero power coefficient. For comparison the corresponding curves for axial oscillations (*1*) are included in Figs. 2 and 3 as dashed lines, where, for a cylindrical reactor with a core diameter equal to the core height, the corresponding value of H^2/M^2 is four times that of R^2/M^2. We see that, for such a reactor, more buckling must be added to excite the first radial harmonic than to excite the first axial harmonic, so that the threshold flux for radial oscillation lies above that for an axial oscillation.

Figure 4 shows the oscillation period at the threshold as a function of the threshold flux level. As shown in Eq. (11) of ref. 1, this period is a function of the threshold flux only and not directly a function of the pile size, flatness, or oscillation mode. Thus the curve of Fig. 4 is good for any degree of flatness and for both axial and "tilt" oscillations.

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D. RANDALL
D. S. ST. JOHN

Savannah River Laboratory
E. I. du Pont de Nemours & Co.
Aiken, South Carolina

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The Transfer Cross Section Between Overlapping Thermal Groups

A neutron thermalization problem which has received some attention in the literature (*1, 2*) is that of finding the variation in space of the thermal neutron spectrum in two adjoining regions of scattering material at different temperatures T and T_0. The overlapping thermal group theory (*2*) approaches this problem by assuming that the neutron flux spectrum has the form

$$\Phi(x, v) = \phi(x)M(v) + \phi_0(x)M_0(v) \tag{1}$$

where v is the velocity, x is the space coordinate, and M and M_0 are maxwellian distributions at the bulk temperatures T and T_0 of the two media. Then the thermalization problem becomes one of solving two diffusion equations in the neutron groups $\phi(x)$ and $\phi_0(x)$ which are considered monoenergetic for the purposes of diffusion. These equations have been given by Selengut (*2*) and are discussed at the end of the letter. In this model neutron thermalization is replaced by a transfer process which takes the neutrons from one group to the other. Selengut (*2*) has given a "transfer cross section"

$$\sigma_T = \frac{2}{A}\sigma_{fa} \tag{2}$$

for a heavy gas of mass A and free atom cross section σ_{fa}. This letter considers a free gas of any mass A, and derives an expression for σ_T, which is valid once assumption (1) has been made. For the part of the spectrum at temperature T_0 in the medium at temperature T it is

$$\sigma_T(A, T_0 \to T) = \frac{2}{A}\sigma_{fa}\frac{\sqrt{1 + T/(AT_0)}}{(1 + 1/A)^2}. \tag{3}$$

In the medium at temperature T, the first part of the spectrum in Eq. (1) does not exchange energy with the medium since it is also at temperature T. However the second part of the spectrum in Eq. (1) does exchange energy because it is at the different temperature T_0. In the overlapping group theory the second group is transferred to the first group in such a way that the rate of change of energy is made equal to that calculated in free gas theory. In a free gas at temperature T which has a scattering cross section σ_s, the rate of change of average energy per neutron of a neutron spectrum which is a maxwellian at temperature T_0 is

$$\frac{\overline{dE_0}}{dt} = -n\int \sigma_s(E_0)\langle E_0 - E\rangle_{av}\, v_0 M_0\, dv_0 \tag{4}$$

where n is the number of atoms per cubic centimeter, $\langle E_0 - E\rangle$ is the average energy change undergone by a neutron of energy E_0 in a collision in a free gas at temperature T, and

$$M_0 = \frac{4}{\sqrt{\pi}}\left(\frac{2kT_0}{m}\right)^{-3/2} v_0^2 \exp\left(\frac{-mv_0^2}{2kT_0}\right) \tag{5}$$

is the fraction of neutrons per unit velocity interval. von Dardel (*3*) has evaluated first the quantity $\langle E_0 - E\rangle_{av}$ and then the integral (*4*) explicitly in the course of his work