those designed for D_2O, and it is suggested that more emphasis be placed on the latter phenomenon. The hydrodynamic instability is a strong function of not only L_B/L_T, but also of the configuration of the test section, and of the entire flow loop, pressure drop, and presumably of power distribution. The use of flow-stabilizing orifices presumably increases the critical L_B/L_T, while the existence of compressible volume in the loop may decrease it; in fact, hydrodynamic instabilities can be induced even in the subcooled region if surge volume is present.

WILLIAM J. LEVEDAHL

Martin-Marietta Corp.
Nuclear Division
Baltimore 3, Md.
Received January 25, 1962
Revised March 27, 1962

Minimum Mass Thin Fins with Internal Heat Generation

In a recent paper (*1*) Minkler and Rouleau have considered the effect of a constant heat generation rate (Q Btu/hr-ft^3) on the heat transfer in thin longitudinal fins. In particular, they have asserted that the temperature gradient is constant in a fin designed to have minimum mass (= minimum profile area A_p) in the class of fins which transfer a specified amount of heat (q_0 Btu/hr-ft) from a base at a specified temperature (T_0) by convection to surroundings at another specified temperature (selected as the zero of the temperature scale) if the thermal conductivity k and the heat transfer coefficient h are constant. I have shown elsewhere (*2*) that this assertion is mathematically incorrect when $Q > 0$ (the error is significant only for large Q), and have set up and solved the problem of Bolza in the calculus of variations to find the fin profile and the temperature distribution for the fin with minimum profile area.

Since the optimum fin profile has a sharp tip (this is true also for the optimum fin with a constant temperature gradient) it is natural to inquire what penalty in profile area must be paid in order to use a fin with a triangular profile, as in Fig. 1. This question is answered here, and the results are presented in Fig. 2, which shows a graph of the dimensionless quantity $\alpha = h^2kT_0^3A_p/q_0^3$ versus $\Lambda = Qq_0^2/kh^2T_0^3$ for the optimum triangular fin as well as for the optimum fin and several points for the optimum fin with a constant temperature gradient. The data for the curved fins are taken from ref. *2*. We also show in Fig. 2 graphs of the dimensionless over-all height of the fin $\mu = whT_0/q_0$.

With reference to Fig. 1, let $q(x)$ and $T(x)$ be respectively the heat flow rate per unit length of fin (Btu/hr-ft) and temperature excess over the surroundings at the point x in the fin where the fin thickness (ft) is $2\delta x/w$. Then

$$q(x) = 2k\frac{\delta x}{w}\frac{dT}{dx}, \frac{dq}{dx} = 2hT - \frac{2Q\delta x}{w}, 0 < x < w, \quad (1)$$

are the differential equations governing the heat transfer in the fin. Since no heat flows through the tip of the fin, we have

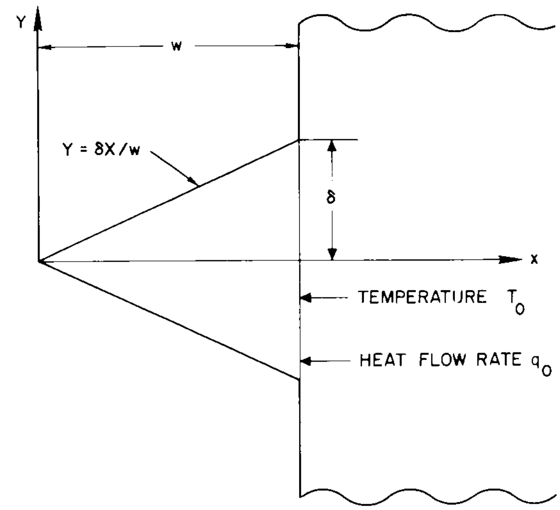

FIG. 1. Sketch of profile of triangular fin

the boundary conditions

$$x = 0, q = 0; x = w, q = q_0, T = T_0. \quad (2)$$

The fin profile area is

$$A_p = \delta w, \quad (3)$$

and the mathematical problem is that of finding two constants δ and w and two functions $q(x)$ and $T(x)$, defined when $0 \leqq x \leqq w$, and satisfying the differential equations (1) and the boundary conditions (2), for which the profile area (3) is a minimum.

If we eliminate q from the differential equations (1) and the boundary conditions (2), we see that

$$\frac{d}{dx}\left(x\frac{dT}{dx}\right) = \frac{hwT}{k\delta} - \frac{Qx}{k}, 0 < x < w, \quad (4)$$

$$x = 0, x\frac{dT}{dx} = 0; x = w, T = T_0, \frac{dT}{dx} = \frac{q_0}{2k\delta}. \quad (5)$$

A particular solution of the inhomogeneous linear differential equation is $Qk\delta^2/h^2w^2 + Q\delta x/hw$, and the general solution can be found by adding to this particular solution the general solution of the homogeneous linear differential equation. If a new variable $u = 2(hwx/k\delta)^{1/2}$ is introduced, the homogeneous equation takes the form

$$u\frac{d^2T}{du^2} + \frac{dT}{du} - uT = 0$$

of Bessel's equation of zero order and imaginary argument. Therefore the general solution of the differential equation (4) is

$$T = \frac{Qk\delta^2}{h^2w^2} + \frac{Q\delta x}{hw} + T_0[BI_0(u) + CK_0(u)], \quad (6)$$

in which B and C are arbitrary constants, and I_0 and K_0 are the standard Bessel functions of zero order. In terms of the dimensionless quantities α, Λ, and μ introduced earlier and $\lambda = 2(hw^2/k\delta)^{1/2}$, we see that

$$Q = \frac{kh^2T_0^3\Lambda}{q_0^2}, w = \frac{\mu q_0}{hT_0}, \delta = \frac{4q_0^2\mu^2}{hkT_0^2\lambda^2},$$

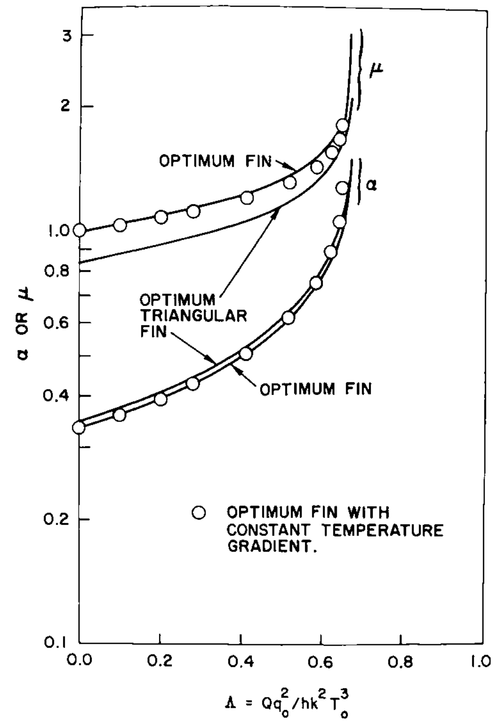

FIG. 2. Graphs of dimensionless minimum profile area $\alpha = h^2kT_0^3A_p/q_0^3$, and dimensionless fin height $\mu = whT_0/q_0$ for (i) optimum fin, (ii) optimum fin with constant temperature gradient, and (iii) optimum triangular fin.

and consequently we find that the function T defined in (6) will satisfy the boundary conditions (5) if and only if $C = 0$,

$$BI_1(\lambda) = (\lambda/4\mu) - (8\mu^2\Lambda/\lambda^3),$$

$$16\mu^3\Lambda[2\lambda I_0(\lambda) - (\lambda^2 + 4)I_1(\lambda)] + 4\mu\lambda^4 I_1(\lambda) = \lambda^5 I_0(\lambda). \quad (7)$$

Thus, the minimizing problem has been reduced to finding, for given Λ, a pair of values λ and μ satisfying (7) for which

$$\alpha = 4\mu^3/\lambda^2 \quad (8)$$

is a minimum. It follows from (8) and $d\alpha/d\lambda = 0$ that $d\mu/d\lambda = 2\mu/3\lambda$. If this value is substituted into the equation obtained by differentiating (7) with respect to λ, the result can be manipulated to read as follows:

$$16\mu^3\Lambda[(2\lambda - \lambda^3)I_0(\lambda) - (\lambda^2 + 4)I_1(\lambda)] + (4\mu\lambda^4/3)[11I_1(\lambda) + 3I_0(\lambda)] = \lambda^5[5I_0(\lambda) + \lambda I_1(\lambda)]. \quad (9)$$

Equations (7) and (9) are a pair of simultaneous linear equations in the variables μ and $\mu^3\Lambda$ and their solution is

$$\mu = \frac{3\lambda[(4\lambda + \lambda^3)I_1^2(\lambda) + 2(8 + \lambda^2)I_0(\lambda)I_1(\lambda) - (8\lambda + \lambda^3)I_0^2(\lambda)]}{8[4(4 + \lambda^2)I_1^2(\lambda) - 2\lambda I_0(\lambda)I_1(\lambda) - 3\lambda^2 I_0^2(\lambda)]}, \quad (10)$$

$$\Lambda = \frac{\lambda^5[3\lambda I_1^2(\lambda) + 4I_0(\lambda)I_1(\lambda) - 3\lambda I_0^2(\lambda)]}{32\mu^3[4(4 + \lambda^2)I_1^2(\lambda) - 2\lambda I_C(\lambda)I_1(\lambda) - 3\lambda^2 I_0^2(\lambda)]}. \quad (11)$$

Equations (8), (10), and (11) can now be used to calculate the values of α and μ plotted versus Λ on Fig. 2 for the triangular case. The maximum possible value of Λ is 0.6640, attained when $\lambda = 4.975$ and in this extreme case the dimensionless profile area α is 1.4746. It is seen that for small and moderate values of Λ the triangular fin is slightly inferior to the optimum fin, as well as to the optimum fin with constant temperature gradient, the inferiority decreasing with increasing Λ. For larger values of Λ the optimum triangular fin becomes superior to the optimum fin with constant temperature gradient and has an area almost indistinguishable from that of the optimum fin.

REFERENCES

1. W. S. MINKLER AND W. T. ROULEAU, The effects of internal heat generation on heat transfer in thin fins. *Nuclear Sci. and Eng.* **7**, 400–406 (1960).
2. J. E. WILKINS, JR., Minimum mass thin fins and constant temperature gradients. *J. Soc. Ind. and Appl. Math.* **10**, 62–73 (1962).

J. ERNEST WILKINS, JR.

John Jay Hopkins Laboratory for Pure and Applied Science, General Atomic Division of General Dynamics Corporation, San Diego, California

Received March 5, 1962
Revised May 22, 1962

Xenon Spatial Oscillations*

In an earlier article (*1*), the threshold values of the flux for oscillations in the axial power distribution were estimated as a function of reactor size and degree of flux flattening, for a cylindrical reactor with a zero power coefficient. The purpose of this letter is to make available the results of more recent calculations that show the effect of nonzero power coefficients, and the threshold values for "flux tilt" oscillations in a cylindrical reactor with a flattened radial power distribution.

The mathematical development and the constants employed in the calculations are given in ref. *1*. In that paper it was shown that the flux threshold for oscillations and the corresponding oscillation periods could be obtained, with an error of less than 5%, from the amount of material buckling that must be added uniformly to the critical reactor to excite the first spatial harmonic orthogonal to the unperturbed power distribution. The relations among the threshold flux level for axial oscillations, the oscillation period, the additional buckling required to excite the first harmonic, μ_1^2, and the power coefficient are given in Eqs. (10) and (11) of ref. *1*.

Figure 1 shows the effect of a nonzero temperature coefficient (*2*) on the flux threshold for xenon oscillations for an unflattened (cosine) power distribution in a slab reactor (or in the axial direction of a cylindrical reactor). The units of the temperature coefficient are milli-k of reactivity per unit power where unit power corresponds to a flux level of 6×10^{13} n/cm^2 sec. For other flux levels, if an effective temperature is defined as being directly proportional to the flux level, the temperature coefficient per unit effective temperature is assumed constant.

* The information contained in this article was developed during the course of work under contract AT(07-2)-1 with the U. S. Atomic Energy Commission.