Consider a system of orthogonal coordinates denoted (q_1, q_2, q_3) or, simply, (q). Let

$$\frac{\partial}{\partial q_i}\boldsymbol{r}(q) = h_i(q)\hat{e}_i(q) \tag{1}$$

and

$$\frac{\partial}{\partial q_i}\hat{e}_i = \frac{1}{h_i}\sum_{k=1}^{3}\Gamma_k^{ij}(q)\hat{e}_k(q) \quad . \tag{2}$$

Then,

$$\Gamma_k^{ij} = \frac{1}{2h_k}\left(\frac{\partial}{\partial q_i}g_{jk} - \frac{\partial}{\partial q_k}g_{ji}\right) \tag{3}$$

with $g_{ij} = \delta_{ij}h_j^2$. The $\hat{e}_i$ forms the local base system; the h_i gives the differential of length $\overline{ds}^2 = \sum_j h_j^2\,\overline{dq_j^2}$. The three-index symbols Γ_k^{ij} are defined in a manner slightly different from the Christoffel symbols of tensor analysis.[2] Their expression in terms of h_j^2 is easily derived. [For example, compare $\frac{\partial^2}{\partial q_i}\frac{\boldsymbol{r}}{\partial q_j}$ with $\frac{\partial^2}{\partial q_j}\frac{\boldsymbol{r}}{\partial q_i}$. Then, note that $\frac{\partial}{\partial q_j}(\hat{e}_i\cdot\hat{e}_k) = 0$.]

Once Eqs. (1), (2), and (3) are accepted, the streaming term can be evaluated effortlessly. We wish to express $\boldsymbol{v}\cdot\frac{\overline{\partial}}{\partial \boldsymbol{r}}f(\boldsymbol{r},\boldsymbol{v})$ (where the bar reminds that $\boldsymbol{v}$ is to be held constant) in terms of derivatives $\frac{\partial}{\partial q_i}$, all $v_j \equiv (\boldsymbol{v}\cdot\hat{e}_j)$ held constant, and derivatives $\frac{\partial}{\partial v_i}$, all q_j held constant. We then have

$$\boldsymbol{v}\cdot\frac{\overline{\partial}}{\partial\boldsymbol{r}}f(\boldsymbol{r},\boldsymbol{v}) = \boldsymbol{v}\cdot\frac{\overline{\partial}}{\partial\boldsymbol{r}}f_1[q,\boldsymbol{v}\cdot\hat{e}_1(q),\boldsymbol{v}\cdot\hat{e}_2(q),\boldsymbol{v}\cdot\hat{e}_3(q)] \tag{4}$$

$$= \frac{v_i}{h_i}\left(\frac{\partial}{\partial q_i} + \Gamma_k^{ji}\frac{v_k}{h_j}\frac{\partial}{\partial v_j}\right)f_1(q,v) \tag{5}$$

$$= \frac{v_i}{h_i}\frac{\partial}{\partial q_i}f_1 + \frac{v_k}{h_k}\left(v_k\frac{\partial}{\partial q_i}h_k - v_i\frac{\partial}{\partial q_k}h_i\right)\frac{1}{h_i}\frac{\partial}{\partial v_i}f_1 \quad , \tag{6}$$

and that is the end of the calculation. [We use the summation convention in Eqs. (5) and (6).]

As an example, we evaluate Eq. (6) for the torus discussed by Pomraning and Stevens.[1] The coordinates are similar to those of the right circular cylinder. One has a pair of plane polar coordinates $q_2 = \rho$, $q_3 = \theta$, $h_2 = 1$, and $h_3 = \rho$ and a coordinate $q_1 = \theta_1$ (rather than $q_1 = z$), which locates the circular section. Corresponding to q_1 is $h_1 = R + \rho\sin\theta \equiv \rho_1$, where R is the radius of the axis of the torus. Then, Eq. (6) becomes

$$\begin{aligned}\boldsymbol{v}\cdot\frac{\overline{\partial}}{\partial\boldsymbol{r}}f &= \frac{v_1}{\rho_1}\frac{\partial}{\partial\theta_1}f_1 + v_2\frac{\partial}{\partial\rho}f_1 + \frac{v_3}{\rho}\frac{\partial}{\partial\theta}f_1\\ &- \frac{v_1}{\rho_1}(v_2\sin\theta + v_3\cos\theta)\frac{\partial}{\partial v_1}f_1 + \left(\frac{v_1^2}{\rho_1}\sin\theta + \frac{v_3^2}{\rho}\right)\frac{\partial}{\partial v_2}f_1\\ &+ \left(\frac{v_1^2}{\rho_1}\cos\theta - \frac{v_2v_3}{\rho}\right)\frac{\partial}{\partial v_3}f_1 \quad .\end{aligned} \tag{7}$$

The transition to the right circular cylinder is achieved by setting $\frac{1}{\rho_1}\frac{\partial}{\partial\theta_1} = \frac{\partial}{\partial z}$ in the first group of terms and neglecting all terms containing ρ_1 in the second group.

An interesting special case occurs when the speed of the particle is fixed. Then, one of the three components of velocity can be eliminated. For example, introduce the variables (v,η,ξ) through $v_1 = v\cos\eta$, $v_2 = v\sin\eta\cos\xi$, and $v_3 = v\sin\eta\sin\xi$. Then, the second group of terms in Eq. (7) becomes $(v = 1)$

$$\frac{\cos\eta}{\rho_1}\sin(\theta+\xi)\frac{\partial}{\partial\eta}f_2(q,\eta,\xi) + \left[\frac{\cot\eta}{\rho_1}\cos\eta\cos(\theta+\xi) - \frac{\sin\eta}{\rho}\sin\xi\right]\frac{\partial}{\partial\xi}f_2(q,\eta,\xi) \quad .$$

These should be compared with Eq. (30) of Ref. 1, after a typographical error has been corrected.

I am grateful to Jeffrey Smith for catching an irritating algebraic error and to G. C. Pomraning for helpful correspondence.

Noel Corngold

California Institute of Technology
Division of Engineering and Applied Science
Pasadena, California 91109

January 2, 1975

[2]See, for example, E. MADELUNG, *Die Mathematischen Hilfsmittel des Physikers*, Dover Publications, Inc., New York (1943).

Corrigendum

M. MARTINI, G. PALMIOTTI, and M. SALVATORES, "A Benchmark Experiment of Neutron Propagation in Iron Used to Test ENDF/B Cross-Section Data," *Nucl. Sci. Eng.*, **56**, 427 (1975).

The second sentence of the Conclusions should read as follows:

The results so far obtained show good agreement between calculation and experiment when the ENDF/B-I data or the more recent data based on an ORNL evaluation (MAT 4180 Mod. 1) are used with proper accounting of the manganese-impurity background effect.