Letters to the Editor

Comments on Variational Theory and Generalized Perturbation Methods

Reference 1 gives an interesting derivation of perturbation expressions relevant to ratios of functionals linear or bilinear with neutron fluxes. In the same paper Stacey compares the formulations obtained with those first derived by Usachev,[2,3] also developed by this author,[4–6] and known under the (more or less accepted) conventional term as "generalized perturbation method" formulas. In one of his final statements, Stacey concludes (p. 455) that "the variational estimates are generally superior to the generalized perturbation estimates, particularly when system alterations with substantial reactivity effects are involved,"[7] apparently (and I quote) because "the assumption of zero reactivity effect for the system alteration being intrinsic to the derivation" of the latter, which instead does not apply to the former. Such a conclusion does not seem correct in many respects. In fact, in problems concerning situations leading to noncriticality, when adopting the "generalized perturbation expressions," the solution was implied, though admittedly not developed, by this author in Ref. 5, where it is stated that in such cases a return to criticality could "be done implicitly recalling the definition of reactivity, which gives $\delta k/k$ in terms of an equivalent change[8] in $\delta\nu/\nu$. Such change (with opposite sign) may be . . . added implicitly to the perturbation." At that time we felt satisfied with the consistent improvement of previous perturbation techniques made possible by the proposed method as far as the flux alteration was concerned, and we did not pay much attention to developing this argument further. Another proof that the "generalized perturbation theory" can also deal properly with noncritical situations was presented by Seki[10] who explicitly extended the formalism of the generalized perturbation method to such cases by introducing the eigenvalue λ ($= 1/k$) in the formalism.[11]

It seems appropriate here to stress a further point that appears to be a more fundamental matter and that results independently from any consideration of merit of the two theories referred to above. Apart from the singular (although not irrelevant) case of the self-perturbation effect of a material insertion (or removal) into (or from) a system, to which the arguments discussed previously mainly apply, we cannot envisage a single experimental situation in which criticality, in one way or another, should not be reestablished by some corresponding change of the system itself.[12] In some cases the change can induce a direct perturbation—i.e., one not through the flux change—of the functional under analysis. Consider a few examples:

1. *Breeding ratio.* If we alter the system we should keep in mind that criticality must be preserved by another corresponding change, such as the fuel enrichment, core size, etc. All such changes should then be considered as producing the perturbation and, therefore, as affecting the breeding ratio. For example, if the first alteration is

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[1]W. M. STACEY, Jr., *Nucl. Sci. Eng.*, **48**, 444 (1972).

[2]L. N. USCHEV, *J. Nucl. Energy*, A/B, **18**, 571 (1964).

[3]L. N. USCHEV and S. M. ZARITSKY, *Atomizdat*, *2*, 242, Bjul. inf. Centr jad. Dannym, Moscow (1965).

[4]A. GANDINI, "A Perturbation Method for Analysis of Neutron Source Experiments," *Proc. I Italian-Polish Seminar Reactor Physics*, Warsaw (Oct. 1966) [See also *Nucl. Sci. Eng.*, **30**, 448 (1967)].

[5]A. GANDINI, *J. Nucl. Energy*, **21**, 755 (1967).

[6]A. GANDINI, *Nucl. Sci. Eng.*, **35**, 141 (1969).

[7]It appears pertinent here to make a distinction between the terms "variational theory" and "generalized perturbation theory" which the author uses, since they can result in a misunderstanding in some instances. Obviously, by "generalized perturbation theory," the author refers to the method used to obtain what he calls "generalized perturbation expressions." But it should be pointed out that so far the term "generalized perturbation" has been adopted mainly to indicate methods (for the sake of the user, and without any specific reference to the procedure followed to arrive at them) for calculating integral parameters so far not amenable to standard perturbation calculation (as, for example, the breeding ratio), and for evaluating with higher accuracy those quantities (such as reactivity worths) that would be treated only to first order by available perturbation expressions: therefore, of more "generalized" scope. On the other hand, the term "variational" clearly refers specifically to the method adopted for the derivation of the perturbative expressions.

[8]That is, starting from a critical system, $\Delta k/k$ of the sample results from considering an altered (equivalent, still critical) one affected by the induced sample perturbation plus a corresponding $\delta\nu/\nu$ change to maintain criticality (see, for example, p. 193, ff of Ref. 9).

[9]A. GANDINI, "Elements of Fast Reactor Physics and Calculations," RT/FI (72)47, CNEN (1972).

[10]Y. SEKI, *Nucl. Sci. Eng.*, **51**, 243 (1973).

[11]In Seki's work only self-perturbation (due to a sample perturbation ΔP) effects are considered and not different alterations (δP) on the system, possibly influencing the perturbation, ΔP, itself [i.e., by a change $\delta(\Delta P)$]. Aside from the fact that, as shown shortly, the self-perturbation effect represents the only case in which, in all foreseeable situations regarding these methods, criticality can be altered, these effects can be dealt with following Seki's formalism and allowing properly for all the direct effects on the functional under consideration.

[12]This does not at all mean that there is no interest in developing perturbation methods regarding noncritical situations but that, in this case, the problems involved should be distinguished from those implied by the author and be directly related to the very time behavior of a reactor system. For example, as considered in Ref. 6, it might be of interest to study the effect of a system alteration on the reaction rate or the reactor power at a given time after the alteration took place. Or, as considered by Komata,[13] the time-integrated reaction rates (or rate ratios) after a given time interval may also be of interest.

[13]M. KOMATA, *Nucl. Sci. Eng.*, **47**, 489 (1972).

represented by a change of a cross section, σ_k, the sensitivity of the breeding ratio, BR, to it will be

$$\frac{d(BR)}{d\sigma_k} = \frac{\partial(BR)}{\partial\sigma_k} + \frac{\partial(BR)}{\partial P_c}\frac{\partial P_c}{\partial\sigma_k} \ , \tag{1}$$

where P_c represents the parameter chosen for reestablishing criticality.

2. *Cross-section adjustment.* As is well known, this represents an important and wide application of the generalized perturbation methods, since they allow the calculation of the sensitivities of the various integral parameters to the cross sections. With these adjustments, the cross sections are forced to become statistically consistent with a variety of integral parameters: reaction rate ratios, reactivity worths, prompt neutron lifetimes, etc. An important parameter that obviously should be included is represented by the (measured) system reactivity, in the sense that the perturbations inherent to all the cross-section adjustments should total a zero contribution. In fact, all these measurements were made on *critical* facilities and, therefore, all the cross-section changes should be forced so as to maintain criticality, within the experimental errors, if the adjusted values are to be consistent with the experimental evidence.

3. *Reactivity worths.* In this case, the generalized perturbation methods can successfully be applied to evaluate changes induced in a reactor system by an alteration δP affecting a reactivity worth, as given by the ratio

$$\rho = \frac{\langle\phi^* \Delta P \phi'\rangle}{\langle\phi^* F' \phi'\rangle} \ , \tag{2}$$

without being forced to recalculate ϕ' for each altered system[14] [easily calculable direct effects of the perturbation δP on ΔP or on F' of Eq. (2) are not considered here]. Here again we meet the requirement of maintaining criticality. In fact, rather than the reactivity value itself, the designer needs ultimately to know, in an accident analysis, the evolution of a given sequence of events in a particular unaltered system and the evolution of the same sequence after alterations (of temperature, composition, etc.) have been introduced. So that the comparison among these cases has sense, the sequence of events and the starting conditions must be the same.[15] Therefore, after evaluating a given sequence of events (for instance a sodium voiding) in an unaltered system, evaluation should be made of the same sequence in the system affected by a given alteration (with respect to temperature, fuel composition, etc.) recognizing the requirement that such alteration maintain criticality under steady state conditions (i.e., at times immediately preceding the initiation of the sequence itself). Merely evaluating the effect on the reactivity of a sodium void by, say, a different fission cross section of ^{239}Pu does not, in principle, make much sense if we do not give due consideration to the fact that such a different cross section implies itself an altered critical system (for instance, with different fuel enrichment or size to maintain criticality). Such alterations should then also be included in the perturbation to give to the reactor designer a proper value of the sodium worth.

[14]More precisely, these generalized perturbation methods give an estimate corresponding to a change $\delta\phi$ rather than $\delta\phi'$ with ϕ' of Eq. (2) replaced by ϕ. [The change $\Delta\phi = (\phi' - \phi)$, due to the self-perturbation effect, may have been accounted for separately by the same methods, as previously described.] This amounts to neglecting second-order effects on the flux.

[15]Apart, of course, from the alteration itself.

4. *Reaction rate ratios.* This case is similar to those discussed above and the conclusions are identical. These measurements are made on critical reactors, and if we need to know the effect of changes on their calculated values resulting from system alterations, these should, in any case, not alter the criticality of the system.

All the examples suggested in Ref. 1 for application of these perturbation methods fall within the above-described cases. To further clarify this important point, consider again, more closely, the relevant case of the breeding ratio. In this event the character of the adjustment necessary to reestablish criticality can significantly change the results.[16] If, for example, the design implies that a different fuel enrichment should be specified in case criticality was badly calculated because of, say, a rather inaccurate plutonium fission cross section, the impact on the breeding ratio of changing such a parameter (in a project analysis survey) will be quite different than in the case where a core size change is foreseen in the same circumstance. In fact, an enrichment change would imply, above all, a strong direct effect on the internal breeding ratio, the ratio of fissile to fertile materials in the core involved. A size change would imply mostly changing the respective contributions from the internal and external breeding ratios to the total one.

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[16]Many practical survey studies are made by theoreticians without a particular reactor project in mind for which an assigned criticality readjustment is specified on technical or economical bases. The analysis can be of an unidentified conceptual reference system and the readjustment can become problematical. In these cases one should assume a set of reasonable hypotheses and consider all of them in the analysis. An approach of this kind was followed, for example, in Refs. 17 and 18 in relation to the breeding ratio.

[17]A. GANDINI, "Study of the Sensitivity of Calculations for Fast Reactors Fueled with Pu^{239}-U^{238} and U^{233}-Th to Uncertainties in Nuclear Data," ANL-6608, Argonne National Laboratory (1962).

[18]A. GANDINI, M. SALVATORES, G. SENA, and I. DAL BONO, "Analysis of Fast Reactors by the CIAP and GLOBPERT Codes Using Improved Perturbation Methods," p. 304, ANL-7320, Argonne National Laboratory (1966).

Response to "Comments on Variational Theory and Generalized Perturbation Methods"

Mr. Gandini argues that it is appropriate in perturbation theory to use a formalism in which the eigenvalue is unchanged because a compensating perturbation must be made to maintain criticality. However, the appropriate formalism depends on just what question is being posed. Mr. Gandini gives several examples of one type of question—if one has a fixed reference case, has good reason to believe his reference calculation is correct, and wants to know the effect of some physical change that would require compensation, then it is appropriate to use a formalism in which the net reactivity worth of the perturbation plus compensation is zero. In this case, the δk terms could be omitted in the variational formalism, or they could be