Reactor Dynamics with Reactivity Loss Rate Proportional to Energy

Soodak (*1*) has considered the dynamic behavior of a nuclear reactor when an accident occurs which produces instantaneously an excess reactivity ρ_0 and there is a (natural or man made) control mechanism serving to remove reactivity at a rate proportional to the energy generated after the accident. Under the assumption that the delayed neutrons have a negligible effect, he derived a third-order, nonlinear, differential system from which the performance of the reactor could be deduced. If the control mechanism is not ultimately disabled the reactivity will decrease steadily from its maximum value ρ_0, becoming zero at some time τ after the accident, and the reactor will be quickly shut down after τ as the reactivity continues to decrease, becoming an approximately linear function of time. In our model the limiting value of the reactivity is $-\infty$, but this simply reflects the fact that the model has lost its utility for very large values of the time. In this paper we shall analyze the differential equation in order the determine the time τ at which the reactor becomes critical again, the energy $Q(\tau)$ generated up to time τ, the power $P(\tau)$ at time τ, which is the maximum power attained after the accident, and the total energy $Q(\infty)$ generated. (Since almost all of the energy is generated in a time comparable to 2τ or 3τ, the failure of our model for very large values of the time should not significantly affect the value of the total energy $Q(\infty)$.) The results are shown in graphical form in Fig. 1.

The accident situation contemplated here has been considered (*2*, *3*) in connection with large insertions of reactivity in a fast reactor. The energy generated as the power rises causes a reactivity drop due to relative motion of the reactor parts.

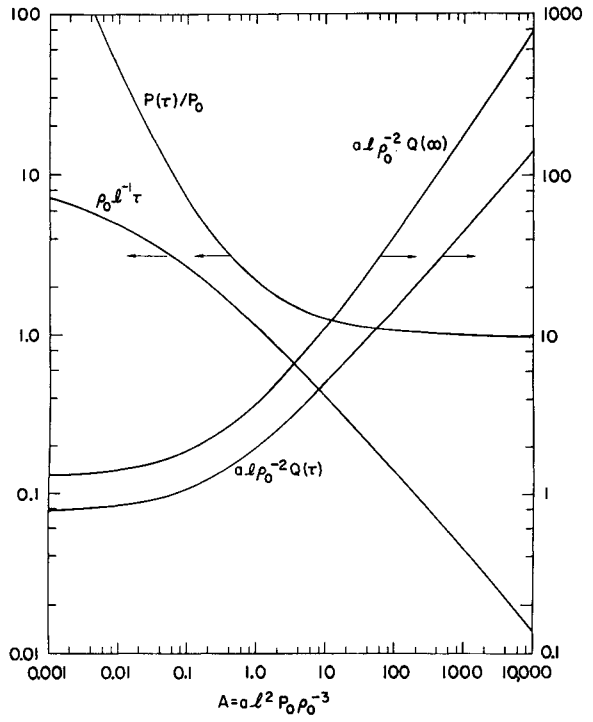

FIG. 1. Graphs of dimensionless time $\rho_0 l^{-1}\tau$ at which reactor becomes critical again, dimensionless energy $al\rho_0^{-2}Q(\tau)$ generated up to that time, ratio $P(\tau)/P_0$ of maximum power to initial power, and total dimensionless energy $al\rho_0^{-2}Q(\infty)$ generated in accident, all expressed as functions of the dimensionless variable $A = al^2P_0\rho_0^{-3}$.

The mathematical statement of the reactor dynamics problem is identical to the mathematical description of the fluid dynamics problem of determining the laminar flow of a fluid over a flat plate when blowing or suction occurs (*4*). The existing tables of solutions to the second problem thus enable the preparation of the graphs of Fig. 1. In addition to this purely numerical work, we shall also derive series expansions for the four quantities graphed in Fig. 1 which can be used for sufficiently large (i.e., $>\frac{1}{2}$) values of the dimensionless parameter $A = al^2P_0\rho_0^{-3}$. Here l is the prompt neutron lifetime, P_0 is the reactor power level at the time of the accident, and a is the energy coefficient of reactivity.

Let $\rho(t)$ be the reactivity at time t after an accident occurs which produces an instantaneous reactivity ρ_0. If $Q(t)$ is the energy released up to time t after the accident, then our hypothesis about the control mechanism is that $d\rho/dt = -aQ$, for some positive constant a. The energy $Q(t)$ is related to the power $P(t)$ so that $dQ/dt = P$, and the assumption that the delayed neutrons are negligible implies that $dP/dt = \rho P/l$. With these differential equations are associated the initial conditions that $\rho(0) = \rho_0 > 0$, $Q(0) = 0$, $P(0) = P_0$. If the energy Q and the power P are eliminated, we see that the reactivity ρ satisfies the third-order, nonlinear, differential system

$$d^3\rho/dt^3 = l^{-1}\rho d^2\rho/dt^2,$$

$$\rho(0) = \rho_0, \quad \rho'(0) = 0, \quad \rho''(0) = -aP_0.$$

Let us now introduce new variables defined so that

$$F = -\alpha\rho/\rho_0, \quad \eta = \rho_0 t/\alpha l, \quad A = al^2\rho_0^{-3}P_0 \tag{1}$$

in which α is a positive constant to be determined. Then

$$d^3F/d\eta^3 + Fd^2F/d\eta^2 = 0,$$
$$F(0) = -\alpha, \quad F'(0) = 0, \quad F''(0) = A\alpha^3. \tag{2}$$

We now choose α as a function of A so that $F'(\infty) = 2$, this choice being motivated by the fact that the differential system, $F''' + FF'' = 0$, $F(0) = -\alpha$, $F'(0) = 0$, $F'(\infty) = 2$, has been used by Emmons and Leigh (*4*) to describe the laminar flow of a fluid across a flat plate when blowing or suction occurs at the plate. The tables of F, F', and F'' in (*4*) may be used as follows to obtain the numerical results for the reactor dynamics problem shown in Fig. 1. For each positive value of α used as an initial value of $-F(0)$ by Emmons and Leigh, there is a corresponding value of $F''(0)$. Then it is readily seen that

$$al\rho_0^{-2}Q(\infty) = 2\alpha^{-2}, \quad A = F''(0)\alpha^{-3}.$$

Moreover, by interpolation, in the tables we can find for each α the value η_0 such that $F(\eta_0) = 0$, and the values $F'(\eta_0)$ and $F''(\eta_0)$. Then the state of the reactor when it has become critical again can be found as follows:

$$\rho_0 l^{-1}\tau = \alpha\eta_0,$$
$$al\rho_0^{-2}Q(\tau) = \alpha^{-2}F'(\eta_0),$$
$$P(\tau)/P_0 = F''(\eta_0)/F''(0).$$

We can also use the tables to trace the history of the reactor by plotting $-\rho/\rho_0$, $al\rho_0^{-2}Q(t) = \alpha^{-2}F'(\eta)$, and $P(t)/P_0 =$

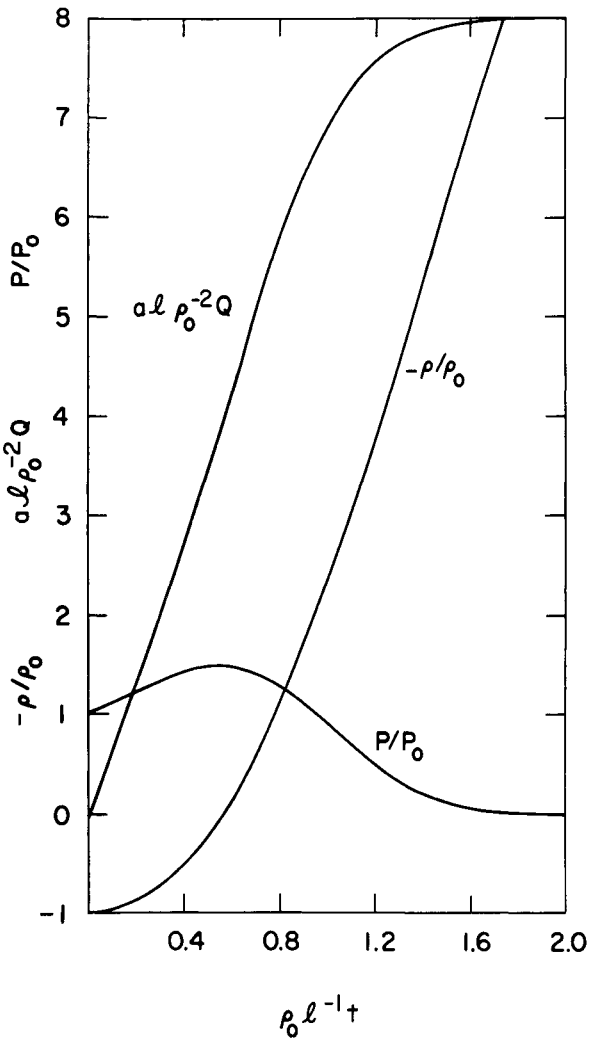

FIG. 2. Graphs of ratio $-\rho/\rho_0$ of negative reactivity to initially inserted reactivity, dimensionless energy $al\rho_0^{-2}Q$ generated, and ratio P/P_0 of power to initial power, as functions of the dimensionless time variable $\rho_0 l^{-1}t$, for an accident for which $A = 5.2637$.

$F''(\eta)/F''(0)$ as a function of $\alpha\eta = \rho_0 l^{-1}t$. This is done in Fig. 2 when $A = 5.2637$, $\alpha = 0.50$.

Let us now return to the system (2) and choose α to be $A^{-1/3}$. A power series of the form

$$F = \sum_{n=0}^{\infty} F_n(\eta)\alpha^n$$

will be a solution of the system (2) provided that

$$F_0''' + F_0F_0'' = 0, \qquad F_0(0) = F_0'(0) = 0, \qquad F_0''(0) = 1,$$

$$F_1''' + F_0F_1'' + F_0''F_1 = 0, \qquad F_1(0) = -1,$$

$$F_1'(0) = F_1''(0) = 0,$$

$$F_n''' + \sum_{r=0}^{n} F_r''F_{n-r} = 0, \; F_n(0) = F_n'(0) = F_n''(0) = 0 \qquad (n \geqq 2).$$

The Runge-Kutta procedure for solving differential equations was applied by Mrs. Marilyn Lindenlaub (née Wagner) on a Datatron 205 to obtain numerical values of F_n, F_n', and F_n'' $(0 \leqq n \leqq 5)$. The numerical integration was continued until the values of F_n' were sensibly constant. In this manner we find that

$$al\rho_0^{-2}Q(\infty) = \alpha^{-2}F'(\infty) = \sum_{n=0}^{\infty} F_n'(\infty)A^{(2-n)/3},$$

$$al\rho_0^{-2}Q(\infty) = 1.65519A^{2/3} + 1.32140A^{1/3} + 0.60900 + 0.14684A^{-1/3} + 0.002469A^{-2/3} - 0.007240A^{-1} + \cdots.$$

Values of $al\rho_0^{-2}Q(\infty)$ calculated from this power series agree satisfactorily (within $\frac{1}{4}\%$) with the numerical values obtained from the Emmons and Leigh tables for A as small as 0.22.

In order to determine the properties of the reactor at the time τ when it has become critical again, we return once more to the system (2), this time supposing that $\alpha = 1$. The function F has a power series expansion

$$F = -1 + \frac{A\eta^2}{2!} + \frac{A\eta^3}{3!} + \frac{A\eta^4}{4!} + \frac{(A - A^2)\eta^5}{5!} + \frac{(A - 5A^2)\eta^6}{6!} + \frac{(A - 16A^2)\eta^7}{7!} + \frac{(A - 42A^2 + 11A^3)\eta^8}{8!} + \frac{(A - 99A^2 + 117A^3)\eta^9}{9!} + \frac{(A - 219A^2 + 735A^3)\eta^{10}}{10!} + \frac{(A - 466A^2 + 3579A^3 - 375A^4)\eta^{11}}{11!} + \cdots, \qquad (3)$$

whose coefficients may be obtained by successively differentiating the differential equation $F''' + FF'' = 0$ and making use of the initial conditions $F(0) = -1$, $F'(0) = 0$, $F''(0) = A$. To solve the equation $F(\eta_0) = 0$, we try a solution η_0 which is a power series in $(2/A)^{1/2}$. If such a power series with undetermined coefficients is substituted into Eq. (3), and similar terms are collected, the first four coefficients can be determined, with the result that

$$\eta_0 = \rho_0 l^{-1}\tau = \left(\frac{2}{A}\right)^{1/2}\left[1 - \frac{3}{20}\left(\frac{2}{A}\right)^{1/2} + \frac{47}{2100}\left(\frac{2}{A}\right) - \frac{11881}{4158000}\left(\frac{2}{A}\right)^{3/2} + \cdots\right].$$

This value of η_0 may now be substituted into the series obtained by differentiating Eq. (3) once and twice, with the result that

$$F'(\eta_0) = al\rho_0^{-2}Q(\tau) = (2A)^{1/2}\left[1 + \frac{4}{15}\left(\frac{2}{A}\right)^{1/2} + \frac{13}{900}\left(\frac{2}{A}\right) - \frac{416}{259875}\left(\frac{2}{A}\right)^{3/2} + \cdots\right],$$

$$\frac{F''(\eta_0)}{A} = \frac{P(\tau)}{P_0} = \left[1 + \frac{2}{3}\left(\frac{2}{A}\right)^{1/2} + \frac{13}{90}\left(\frac{2}{A}\right) + \frac{67}{9450}\left(\frac{2}{A}\right)^{3/2} + \cdots\right].$$

Values of η_0, $F'(\eta_0)$, and $F''(\eta_0)$ calculated from these series agree quite well (within $\frac{1}{2}\%$) with those obtained from the

Emmons and Leigh tables for A as small as 0.53, 0.23, and 0.39, respectively.

The reactor dynamics problem can be slightly generalized to include the effect of a linear reactivity insertion or removal, so that $d\rho/dt = b - aQ$. Instead of the substitution (1) we now suppose that

$$F = -\alpha\rho/\rho_0, \qquad \eta = \eta_1 + (\rho_0 t/\alpha l),$$
$$A = al^2\rho_0^{-3}P_0, \qquad B = bl\rho_0^{-2}$$

in which α and η_1 are constants. Then

$$d^3F/d\eta^3 + Fd^2F/d\eta^2 = 0,$$
$$F(\eta_1) = -\alpha, \qquad F'(\eta_1) = -B\alpha^2, \qquad F''(\eta_1) = A\alpha^3,$$

and the quantities α and η_1 are selected so that $F'(0) = 0$, $F'(\infty) = 2$. Numerical results may be obtained from the tables in (*4*) as follows. For each negative value of $F(0)$ and each η_1 used by Emmons and Leigh, values of $F(\eta_1)$, $F'(\eta_1)$, and $F''(\eta_1)$ can be read from the tables. Then $\alpha = -F(\eta_1)$, $B = -F'(\eta_1)/\{F(\eta_1)\}^2$, $A = F''(\eta_1)/\{F(\eta_1)\}^3$, and the total energy release $Q(\infty)$ is such that

$$al\rho_0^{-2}Q(\infty) = B + 2\alpha^{-2}.$$

Of course η_1 must be restricted to insure that $F(\eta_1) < 0$, in order that $\alpha > 0$. If the value η_0 where $F(\eta_0) = 0$ is found by interpolation in the tables, as well as the values $F'(\eta_0)$ and $F''(\eta_0)$, then the state of the reactor at the time τ when it has become critical again can be found as follows:

$$\rho_0 l^{-1}\tau = \alpha(\eta_0 - \eta_1), \qquad al\rho_0^{-2}Q(\tau) = B + \alpha^{-2}F'(\eta_0),$$
$$P(\tau)/P_0 = F''(\eta_0)/F''(\eta_1).$$

Unfortunately, all the values $F'(\eta_1)$ in the tables are positive, so that only negative values of B can be handled in this manner. Hence, linear reactivity removal problems can be solved, but linear reactivity insertion problems cannot until the tables in (*4*) are extended to negative values of η.

REFERENCES

1. H. Soodak (ed.), "Reactor Handbook, Vol. III A, Physics." Wiley, New York, 1962.
2. R. Daane, R. Mela, and J. De Felice, Progress report on the study of low probability, high hazard, fast reactor accidents. NDA 14-107 (November 1956).
3. C. Klahr, Energy release in high hazard accidents. NDA 14-149 (October 1956).
4. H. W. Emmons and D. C. Leigh, "Tabulation of the Blasius Function with Blowing and Suction." Aeronautical Research Council, CP. No. 157, London (1954).

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Sintering of UO_2 in Carbon Dioxide Atmosphere

As compared with conventional hydrogen sintering, the use of slightly oxidizing atmospheres in the high temperature stage of the UO_2 sintering cycle offers the advantage of reducing both temperature and time necessary to attain a certain fired density.

Among the possible atmospheres of this kind, steam has been widely and thoroughly investigated for many years, and several setbacks have shown up together with the merits, mainly as a consequence of the vicious nature of steam as a furnace atmosphere (*1*, *2*).

Carbon dioxide, on the contrary, which is free of almost all of the disadvantages of steam, has been suggested as a sintering atmosphere (*3*), but, as far as we know, never used. We tried it,[1] in comparison with conventional hydrogen sintering, on a certain brand of depleted UO_2 powder, supplied by the Mallinckrodt Chemical Works, St. Louis, Mo., U.S.A., the properties of which are summarized in Table I. Sintering cycles, both in H_2 and CO_2, are shown in Fig. 1.

Pellets were pressed to about 6 gm/cc from a powder with 2 wt.% polyethylene glycol, 1 wt.% polyvinyl/alcohol and 2 wt.% vegetal stearine. Comparative results in CO_2 and H_2 are shown in Fig. 2, while the influence of time and temperature in CO_2 is shown in Fig. 3. The sintering activation energy, calculated according to Jordan and Duwez (*4*), was 40,300 ± 1300 cal/mol (Fig. 4), to be compared with 32,000 ± 3200 cal/mol for the same powder in steam (*1*) and 76,000 cal/mol in hydrogen (*5*). Whether this discrepancy is due to the inaccuracy of Jordan and Duwez's method or to a basically different mechanism for steam and CO_2 sintering, is not known.

TABLE I

Properties of MCW Ceramic Grade Depleted Powder

Real density (CCl_4), gm/cc	10.34
Tapped density, gm/cc	2.18
Average particle diameter (Fisher), μ	1.28
Total surface area (B.E.T.), $m^2\ gm^{-1}$	3.45
External surface (Blaine), $m^2\ gm^{-1}$	0.42
Roughness factor	8.47
O/U ratio	2.03

The organic additions to the powder were in general deletorious to the final density, as shown in Fig. 5. The properties of the CO_2 sintered pellets (porosity, grain size, etc.) compared very well with those of H_2 and steam sintered pellets of corresponding densities. In particular, the stoichiometry of the final product was insured by the final hydrogen cooling and no carbon pickup was detected.

REFERENCES

1. W. E. Baily, J. C. Danko, H. M. Ferrari, and R. L. Colombo, *J. Am. Ceram. Soc.* **41**, 768 (1962).
2. I. Amato, R. L. Colombo, and A. M. Protti, to be published.

[1] FIAT patent n° F-1534.