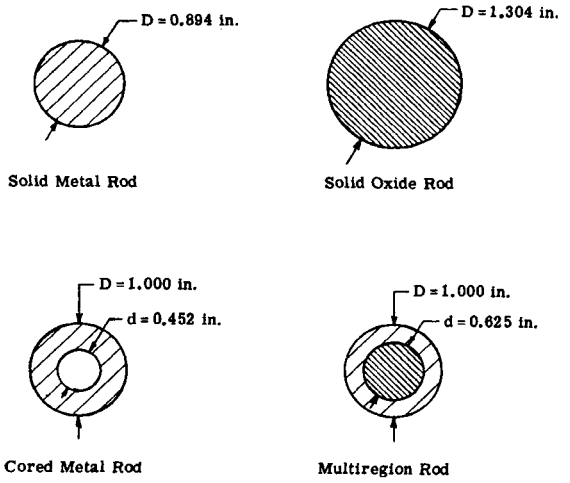

FIG. 4. Fuel element configurations for equal uranium content. Assumptions: Maximum allowable metal temperature = 1100°F. Maximum allowable oxide temperature = 4000°F. Surface temperature = 750°F.

2. From Fig. 1 it is seen that the maximum metal temperature at any given power generation rate is about 65°F lower for the multiregion element than for the cored metal element. Using the data in Fig. 2, this indicates an increase in the maximum permissible burnup of about 25%. Thus a considerable improvement is obtained in the swelling limited burnup.

Calculations have been made to determine the relative burnup of the four elements under consideration when limited by available excess reactivity. With a given total uranium content and a given enrichment—natural or 2% enriched—the burnup limits are the same for all elements except the solid oxide rod. The latter permits 5% to 25% less burnup under these conditions. Thus the multiregion element has no disadvantage with respect to reactivity-limited burnup.

The reactivity of a fuel element in a given lattice for any given enrichment will depend upon the total uranium content of the element. To determine the results of this effect on the multiregion fuel element, calculations were performed to determine the maximum permissible uranium content at a given maximum metal temperature and power generation rate. The results are illustrated in Fig. 3 in comparison with the cored metallic rod. It can be seen that the multiregion element permits from 4% to 20% greater uranium content under otherwise identical operating conditions.

In the course of previous work at United Nuclear, estimates of the effect of uranium content on reactivity were made for similar fuel elements. Then calculations were made specifically for the Sodium-Deuterium Reactor (SDR). Based on these data it is expected that a multiregion element will have between 0.5% and 2.0% greater reactivity than a cored metal element of the same outside diameter, operating at the same power generation rates and maximum metal temperatures. One could take advantage of this gain in three different ways: lower enrichment, smaller core size, or lower refueling frequency if the burnup is reactivity-limited.

It is also necessary to evaluate the effect of element geometry on reactivity to determine whether the above-cited gains are actually available. For this purpose the four fuel elements shown in Fig. 4 have been used. They have equal uranium content. Calculation of the reactivity for an infinite lattice (K_∞) of such elements was made. The multiregion element does not differ appreciably from either the cored or solid metal element and is somewhat superior to the solid oxide element. The conclusions reached above regarding the greater reactivity of the multiregion element because of its greater uranium content are therefore sustained.

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Thermal Flux Disadvantage Factors for Slab Geometry

It is the purpose of this note to point out that thermal flux disadvantage factors for slabs can be calculated accurately by an extremely simple method. This method is based partly on blackness theory.

In 1959, Maynard (*1*) proposed a method for the theoretical calculation of thermal disadvantage factors. This method was based *only* on blackness theory. The DB_1 approximation was reported to give the best results. However, this procedure is not simple for calculational purposes with the presently available tabulated functions.

Recently, Theys (*2*) has given a simple expression for thermal flux disadvantage factors. Theys' treatment is based on the Integral Transport Theory, and on the arguments put forward by Amouyal and Benoist. In Theys' notation, the flux disadvantage factor is given by Eq. (1).

$$\bar{\phi}_1/\bar{\phi}_0 = G + a\Sigma_0^c[\Sigma_1^{tr}(b - a) + 0.13] \qquad (1)$$

where

$$G = \phi_a/\bar{\phi}_0 .$$

Here, $\bar{\phi}_1$ and $\bar{\phi}_0$ are the average fluxes in the moderator region, and the fuel element respectively. ϕ_a is the neutron flux at the surface of the fuel slab. The subscripts 0 and 1 refer to the fuel element and the moderator, respectively. The half thickness of the fuel (absorbing) slab and of the moderator region are denoted by a and $(b - a)$. The fuel disadvantage factor is given by Theys as:

$$G = 1 + \frac{\Sigma_0^c}{\Sigma_0^T}\left[\frac{P_c}{1 - P_c} - 2a\Sigma_0^T\right] \cdot \left[1 + \alpha^*\left(\frac{\Sigma_0^S}{\Sigma_0^T}\right) + \beta^*\left(\frac{\Sigma_0^S}{\Sigma_0^T}\right)^2\right]. \qquad (2)$$

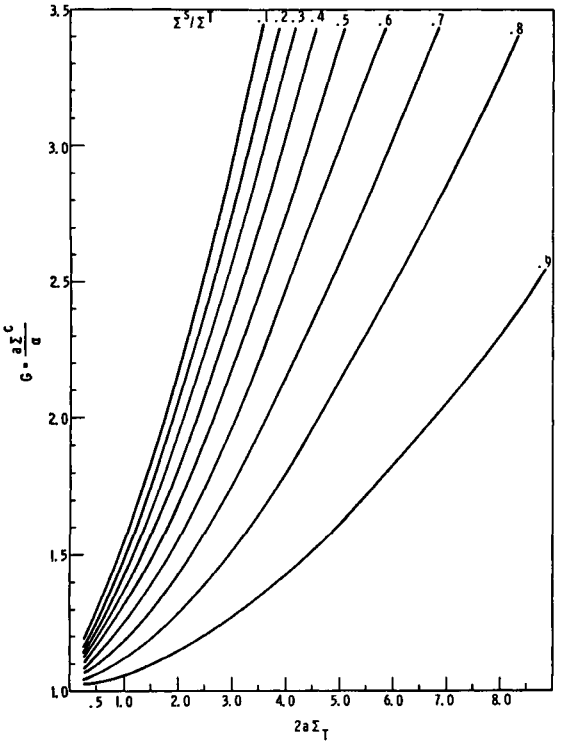

FIG. 1. Fuel disadvantage factor versus product of slab thickness and macroscopic total cross section.

TABLE I

COMPARISON[a] OF FLUX DISADVANTAGE FACTORS COMPUTED FROM THEYS' METHOD, BLACKNESS THEORY AND DSN (S-8) FOR TWO VALUES OF THE HALF THICKNESS OF A SLAB

Σ_0^S/Σ_0^T	$2\,a\Sigma_0^T$	a = 0.5 cm			a = 1.0 cm		
		Theys' method	Blackness theory[b]	S-8	Theys' method	Blackness theory[b]	S-8
0.1	0.25	1.435	1.466	1.475	1.697	1.729	1.728
	0.50	1.821	1.863	1.895	2.345	2.387	2.384
	1.0	2.616	2.660	2.722	3.664	3.708	3.675
0.3	0.25	1.338	1.363	1.371	1.542	1.567	1.565
	0.50	1.639	1.671	1.697	2.047	2.079	2.080
	1.0	2.261	2.294	2.343	3.077	3.110	3.093
0.5	0.25	1.242	1.259	1.266	1.387	1.405	1.403
	0.50	1.458	1.480	1.499	1.749	1.770	1.772
	1.0	1.905	1.927	1.963	2.487	2.509	2.497

[a] In all cases V_M/V_F = 2.0 and Σ_1^{tr} = 2.33 cm^{-1}. The subscript zero refers to the fuel element.

[b] The flux disadvantage factors were computed from Eq. (4).

Without any approximations used in Theys' derivation of the fuel disadvantage factors, we evaluate Theys' Eq. (4) by blackness theory (*3*). We get for G

$$G = a\Sigma_0^c/\alpha, \tag{3}$$

where α, the ratio of the current to the flux at the surface of the fuel slab, is computed by blackness theory. Using Eq. (3) and Theys' argument (for the moderator region), we get the following expression for the flux disadvangage factor,

$$\bar{\phi}_1/\bar{\phi}_0 = a\Sigma_0^c\left[\frac{1}{\alpha} + \Sigma_1^{tr}(b - a) + 0.13\right]. \tag{4}$$

It is obvious from Eq. (4) that, once $a\Sigma_0^c/\alpha$ is known, thermal utilization factors are trivial to calculate. In Fig. 1, a set of curves is given for $a\Sigma_0^c/\alpha$ versus $2a\Sigma_0^T$ for Σ_0^S/Σ_0^T = 0.1 to 0.9 in steps of 0.1. We have computed[1] these curves from Maynard's (*1*) tables. However, α can also be computed from Eq. (5) using the capture fractions F_0 and F_1, as tabulated by Schiff and Stein (*4*)

$$\alpha(a\Sigma_0^c, \Sigma_0^S/\Sigma_0^T) = \frac{F_0}{2[2 - F_1]}. \tag{5}$$

In Maynard's notation (*1*),

$$F_n = 1 - (n + 2)(R_{1n} + T_{1n}).$$

In Table I, the calculations of flux disadvantage factors, based on the proposed method (Eq. 4), are compared with those performed with Theys' method and the S-8 method (*5*). In all the S-8 calculations, the number of mesh intervals was taken to be sixteen and twenty-six for the fuel element and the moderator region. The parameter specifying the convergence criterion, ϵ, was set equal to 10^{-4}. It is clear from Table I that this extremely simple method gives more accurate results than Theys' method. This better agreement with the S-8 calculation arises from the fact that the values of G computed from Eq. (3) are larger (and more accurate) than those calculated from Theys' method. It is to be noted that the only difference between our method and Theys' method is in the evaluation of the fuel disadvantage factor.

The author is thankful to Dr. W. Zernik for a stimulating discussion. Helpful discussions with Dr. C. W. Maynard are also gratefully acknowledged.

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[1] The derivation of Eq. (5) is given by Henry (*3*). Also see A. D. VOORHIS AND T. M. RYAN, Reactor Control meeting held in Los Angeles, March 6-8, 1957. TID-7532 (Pt. 1) p. 206. It should be noted that this expression for α slightly differs from the DB_1 approximation (of Maynard in ref. *1*), which gives $\alpha = F_0/2(2 - F_0)$.

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Slowing Down with Anisotropic Scattering

The technique of expanding Laplace transforms in a series of the transform variable before inversion was applied in an earlier paper (*1*) to determine the flux obtained from an energy distributed neutron source in an infinite homogeneous system. This method is capable of extension to slowing down when the differential scattering cross section of the nuclei is anisotropic in the center of mass system. We will assume that the angular distribution of scattered neutrons is independent of the initial energy and will also omit extraneous complications, such as absorption, in order to give a simple outline of the main feature of the problem.

The angular distribution of scattered neutrons is assumed to be given by $g(\mu)$ where μ is the cosine of θ, the scattering angle in the center of mass system. In terms of the initial and final lethargies u' and u,

$$\mu = \frac{2}{1-\alpha} e^{-(u-u')} - \frac{1+\alpha}{1-\alpha},$$

where α is the maximum fractional energy loss on a collision.

The neutron balance equation in the lethargy interval du at u is

$$\Sigma_s(u)\phi(u) = \int_{u-u_m}^{u} \Sigma_s(u')\phi(u')\, e^{-(u-u')} \cdot g(\mu)\frac{du'}{1-\alpha} + S(u) \tag{1}$$

where $\phi(u)$ is the flux, $\Sigma_s(u)$ the scattering cross section, and $S(u)$ the source, all at lethargy u, and $u_m = \ln(1/\alpha)$. Taking the Laplace transform of Eq. (1) and rearranging terms we find

$$\mathcal{L}\{\Sigma_s(u)\phi(u)\} = \frac{1}{1-K(p)}\,\mathcal{L}\{S(u)\} \tag{2}$$

where

$$K(p) = \int_0^{u_m} e^{-(p+1)u} g\left(\frac{1}{\delta}e^{-u} - \frac{1}{\delta} + 1\right)\frac{du}{1-\alpha} = \frac{1}{2}\int_{-1}^{1}[1-\delta(1-\mu)]^p g(\mu)\, d\mu \tag{3}$$

and $\delta = \frac{1}{2}(1-\alpha)$.

The normalizing condition on $g(\mu)$ is $\int_{-1}^{1} g(\mu)\, d\mu = 2$ which is equivalent to $K(0) = 1$. Thus the expansion of $\{1-K(p)\}^{-1}$ in powers of p is

$$\frac{1}{1-K(p)} = \frac{1}{\xi_A\, p} + \gamma_{0A} + \gamma_{1A}p + \gamma_{2A}p^2 + \cdots$$

where ξ_A and γ_{nA} are constants to be determined.

Inverting Eq. (2) gives

$$\Sigma_s(u)\phi(u) = \frac{1}{\xi_A}\int_0^u S(u')\, du' + \gamma_{0A}\, S(u) + \gamma_{1A}\, S'(u) + \cdots. \tag{4}$$

The coefficients in Eq. (4) are found from the relation

$$\gamma_{nA} = \lim_{p\to 0}\frac{1}{p^n}\left[\frac{1}{1-K(p)} - \frac{1}{\xi_A\, p} - \gamma_{0A} - \gamma_{1A}\, p - \cdots - \gamma_{n-1,A}\, p^{n-1}\right]$$

with

$$\frac{1}{\xi_A} = \lim_{p\to 0}\frac{p}{1-K(p)} = -\frac{1}{K'(0)}.$$

Thus

$$\gamma_{nA}\{K'(0)\}^2 = \frac{K^{(n+2)}(0)}{(n+2)!} - K'(0)\left[\frac{\gamma_{0A}\,K^{(n+1)}(0)}{(n+1)!} + \frac{\gamma_{1A}\,K^{(n)}(0)}{n!} + \cdots + \frac{\gamma_{n-1,A}\,K''(0)}{2!}\right] \tag{5}$$

where from equation (3) we have that

$$K^{(r)}(0) = \frac{1}{2}\int_{-1}^{1}\{\ln[1-\delta(1-\mu)]\}^r g(\mu)\, d\mu. \tag{6}$$

The logarithm in the integrand of (6) can be expanded in powers of δ to any required accuracy, thereby determining the coefficients in Eq. (4) for any scattering material. For all but the lightest nuclei it will be found adequate to retain only the terms of $O(\delta^2)$ in Eq. (6) and the first two terms of the series in Eq. (4).

As an illustration we now treat the simplest problem of a heavy scatterer for which $\alpha \to 1$. We may therefore neglect terms $O(\delta^2)$ and take $\delta = \xi$, the average logarithmic energy decrement for isotropic scattering.

Equation (6) now gives

$$K^{(r)}(0) = (-1)^r\frac{1}{2}\xi^r\int_{-1}^{1}(1-\mu)^r g(\mu)\, d\mu = (-1)^r\xi^r\overline{(1-\mu)^r} \tag{7}$$

where the bar denotes the mean value. Substituting from (7) into (5) and rearranging the terms,

$$\gamma_{nA}\overline{(1-\mu)} - \xi\gamma_{n-1,A}\frac{\overline{(1-\mu)^2}}{2!} + \cdots + (-1)^n\xi^n\gamma_{0A}\frac{\overline{(1-\mu)^{n+1}}}{(n+1)!} = (-1)^n\frac{\xi^n}{(n+2)!}\frac{\overline{(1-\mu)^{n+2}}}{\overline{(1-\mu)}}. \tag{8}$$

For a heavy scatterer the first two coefficients in the expansion (4) are thus

$$\xi_A = \xi(1-\bar{\mu})$$
$$\gamma_{0A} = \frac{1-2\bar{\mu}+\bar{\mu^2}}{2(1-\bar{\mu})^2} \tag{9}$$