Letters to the Editors

Transport Mean Free Path and Diffusion Length of Neutrons Calculated From Markov Chain Statistics*

In a recent paper (*1*), Ferziger *et al.* present a calculation of the diffusion length in a homogeneous medium in terms of a random walk treatment. The result, as the authors belatedly recognized, had been previously given by Fermi (*2*), but has apparently become part of the forgotten literature. It may therefore not be inappropriate to present here an alternative derivation of the transport mean free path and the diffusion length which is somewhat more concise and which gives additional physical insight into the concepts of the random walk mechanism. Frankly, I can not guarantee that *this* derivation is original, and perhaps it also can be traced back to Fermi—it is certainly the sort of exercise that would have been completely in character for him.

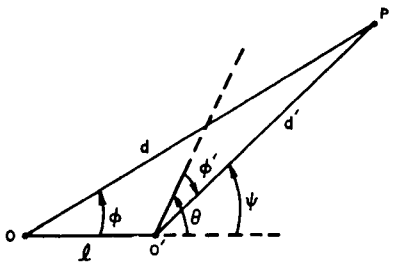

FIG. 1

1. Transport Mean Free Path (*3*). We define λ_{tr} as the mean distance traveled by a neutron projected onto the direction of its original motion. Let a neutron be released at the origin, moving in the direction of the positive x-axis. The transport mean free path is then the mean value of the x coordinate of the neutron at the point of absorption or (in a nonabsorbing medium) after infinitely many collisions. After the neutron has been released it travels a path length of distance l, suffers a collision, and if it is not absorbed, is scattered through an angle θ, to continue its peregrinations. The neutron finally arrives at the point P at distance d and angle ϕ from the origin (Fig. 1). The transport mean free path is then

$$\lambda_{tr} = \langle d \cos\phi \rangle = \langle l \rangle \frac{\sigma_a}{\sigma} + \langle l + d' \cos\psi \rangle \frac{\sigma_s}{\sigma} = \langle l \rangle + \langle d' \cos\psi \rangle \frac{\sigma_s}{\sigma} \tag{1}$$

We may express the angle ψ in terms of θ, the scattering angle, and ϕ', the angle between the scattered direction and final point of absorption. There is in addition an azimuthal angle, but since the scattering is axially symmetric, even though it may not be isotropic, we can immediately write

$$\langle d' \cos\psi \rangle = \bar{\mu} \langle d' \cos\phi' \rangle \tag{2}$$

where $\bar{\mu} = \langle \cos\theta \rangle$ is the mean cosine of the angle of scattering. Since neutron diffusion is a Markov process, $\langle d' \cos\phi' \rangle = \langle d \cos\phi \rangle = \lambda_{tr}$, and hence

$$\lambda_{tr} = \lambda + \bar{\mu}\lambda_{tr}\frac{\sigma_s}{\sigma} \tag{3}$$

whence

$$\lambda_{tr} = \lambda/(1 - \bar{\mu}\sigma_s/\sigma) \tag{4}$$

2. Diffusion Length. The diffusion length is, by standard definition (*4*), $\frac{1}{6}\langle d^2 \rangle$, and we can write

$$\langle d^2 \rangle = \langle l^2 \rangle \frac{\sigma_a}{\sigma} + \langle l^2 + 2l\, d' \cos\psi + d'^2 \rangle \frac{\sigma_s}{\sigma}$$

and since we can write (*2*)

$$\langle l \rangle = \lambda$$

$$\langle l^2 \rangle = 2\lambda^2$$

$$\langle l\, d' \cos\psi \rangle = \langle l \rangle \langle d' \cos\psi \rangle = \lambda\lambda_{tr}\bar{\mu}$$

we have immediately

$$6L^2 = 2\lambda^2 + (2\lambda\lambda_{tr}\bar{\mu} + 6L^2)\sigma_s/\sigma$$

whence

$$L^2 = \frac{\lambda\sigma}{3\sigma_a}\left(\lambda + \bar{\mu}\lambda_{tr}\frac{\sigma_s}{\sigma}\right) = \frac{1}{3}\lambda_a\lambda_{tr} \tag{5}$$

There is an important point which should be made here; the diffusion length defined in Eq. (5) is exact for *all* absorption ratios and for *any* scattering law. It is not equal to the asymptotic diffusion length, $1/\kappa$, except in the usual limit of small absorption; the mean square distance to absorption is $6L^2$, and not $6/\kappa^2$, because of the contributions near the source from the nonasymptotic part of the exact solution of the transport equation.

*Work supported by USAEC under contract number AT-11-1-GEN-8.

REFERENCES

1. J. H. FERZIGER, G. S. C. WANG, AND P. F. ZWEIFEL, *Nuclear Sci. and Eng.* **10,** 285 (1961).
2. E. FERMI, "Nuclear Physics," p. 186. Univ. of Chicago Press, 1949.
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4. A. M. WEINBERG AND E. P. WIGNER, "The Physical

Theory of Neutron Chain Reactors." Univ. of Chicago Press, 1958.

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Received October 11, 1961

Multiregion Fuel Elements

The performance of metallic uranium fuels is limited by the operating temperature at which dimensional instability becomes excessive. Although uranium oxide fuels may be operated at much higher temperatures, they have the disadvantages of low uranium density and low thermal conductivity. A multiregion fuel element can be designed to take advantage of the high density and high thermal conductivity of the metal as well as of the high operating temperature of the oxide. This is accomplished by using a rod-shaped element having an outer annulus of metal and a central core of oxide. The discussion is limited to metallic uranium and uranium oxide; however, the same principles apply to various uranium alloys and refractory uranium compounds. For instance the core may be a high molybdenum alloy, or uranium carbide, instead of uranium oxide; the annulus may be an alloy of uranium instead of unalloyed uranium.

Preliminary analytical evaluation of the relative merits of the following four types of fuel elements have been made: (1) solid metallic uranium rods, (2) solid uranium oxide rods, (3) cored metallic uranium rods, (4) multiregion metallic uranium–uranium oxide rods. The evaluation concluded that the multiregion fuel element provides improved performance over other elements by permitting 20% increase in power generation, or 25% increase in burnup, or 1% increase in reactivity.

Although the potential increase in permissible burnup would make multiregion elements desirable for enriched, compact reactors, it is expected that their greatest utility would be found in natural or low enrichment fuel power reactors, where the increased reactivity is of prime importance.

The method used in the evaluation and the results obtained are discussed below.

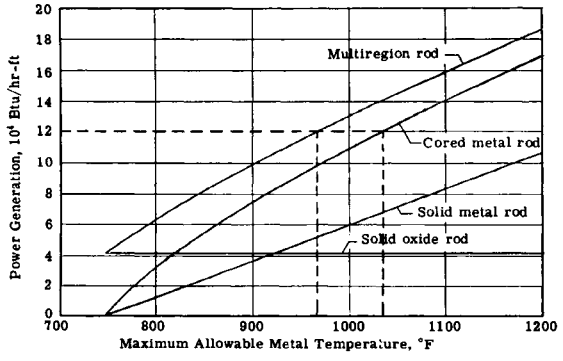

FIG. 1. Fuel element power generation. Assumptions: Equal uranium content in all elements. Equal maximum metal temperature in all elements. Equal maximum oxide temperature in all elements (4000°F). Equal surface temperature in all elements (750°F).

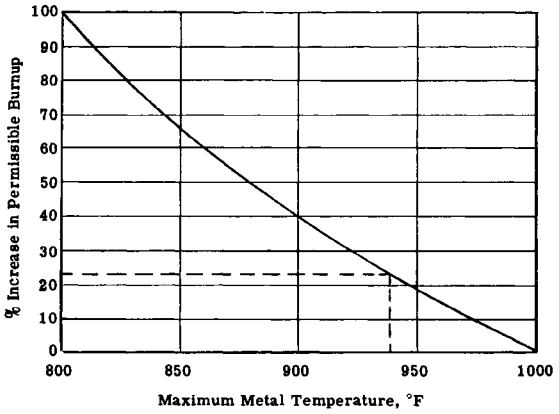

FIG. 2. Burnup as a function of temperature. Assumptions: 15% swelling at 0.5 atom % burnup. ANL swelling data are applicable. Relative burnup at 1000°F is 100%.

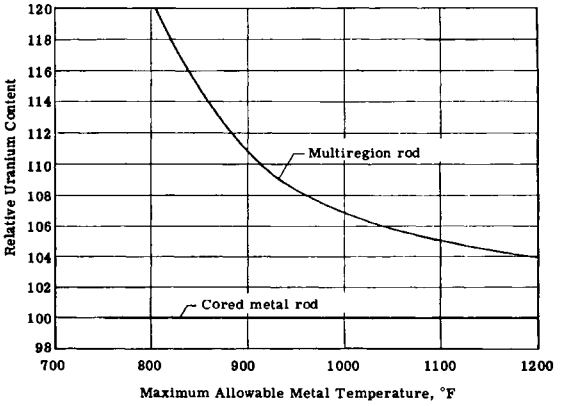

FIG. 3. Fuel element uranium content. Assumptions: Equal power generation in all elements. Equal maximum metal temperature in all elements. Equal surface temperature in all elements (750°F). Maximum oxide temperature of 4000°F.

The results of the permissible power generation evaluation are shown graphically in Fig. 1 along with the assumptions used. The data show that the multiregion element provides greater power generation per unit length than any of the other elements. It represents an improvement of nearly 20% over its nearest competitor, the cored metallic element.

The maximum permissible burnup of a fuel element is limited either by the radiation-induced swelling or by the available excess reactivity; in general, metallic elements are governed by the former while oxide elements are governed by the latter. A comparison was made between the cored metallic element and the multiregion element to determine the relative advantage of the multiregion element with respect to swelling limited burnup. To make quantitative estimates it was assumed that swelling limits the burnup, and that the amount of swelling is a function of maximum metal temperature. Using the Argonne National Laboratory experimental data (*1*) and assuming a permissible swelling of 15% volume increase at 0.5 atom per cent burnup, the allowable relative burnup as a function of temperature was determined. This is shown in Fig.