LETTERS TO THE EDITOR

Regarding "Elastic-Plastic Thermal Stresses in Tubes Subjected to Uniform Heat Generation Evaluation of Experimental Results Obtained Using Graphite Tubes"*

The author has analyzed the case of a cylindrical tube with uniform internal heat generation, with the heat removed symmetrically from the exterior cylindrical surface, and with the axial thermal expansion of the tube completely suppressed. The latter condition is indicated by the author's assumption in his second paragraph that there is no axial strain and is confirmed by Fig. 6 which shows axial compressive stresses over the entire cross section of the tube. In view of the obvious desirability of allowing freedom for axial expansion, the author should have explained why he assumed that axial expansion was suppressed. If a reactor were designed with axial thermal expansion of the fuel elements suppressed, buckling of the fuel elements might be a serious problem, and upon reduction of the heat generation, there might be either a problem of fracture due to tensile stresses caused by suppression of axial thermal shrinkage or a problem associated with shortening of the fuel elements as a result of prior axial plastic compression. This shortening might allow vibration under the action of forces exerted by the reactor coolant.

The writer finds it surprising that the author did not reveal until his penultimate paragraph that the tests which he correlated with his theoretical analysis involved axial restraint which was not complete but which was "believed to be small." The fundamental difference between theoretical and test conditions eliminates all reason for existence of a correlation, and this apparently explains the gross disagreement between theory and experiment indicated by Fig. 2.

It appears reasonable to conclude that the failures which were encountered in the tests were caused by tensile stresses and strains in the graphite near the cooled surface. Fracture in a body subjected to a single application of nonuniform temperature would not occur simply as a consequence of the body developing plastic strains throughout most or even all of its volume. Fracture would be initiated only because local strains produced by thermal differential expansions are sufficiently high that they cannot be accommodated by plastic flow and elastic deformation without elevation of stresses to levels sufficient to cause fracture.

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* By T. Kammash, *Nuclear Sci. and Eng.* **7**, 425–434 (1960).

Rebuttal

Mr. Miller's explanation of the failure of the tubes is simply a restatement of the general theory of failure. In effect, it states that fracture occurs when the stresses in the material exceed a certain level. I agree wholeheartedly.

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Slowing-Down Time of Neutrons in Water

The slowing-down time, t_s, of high-energy neutrons to the cadmium cutoff energy in a moderator having $1/v$ absorption may be obtained from a measurement of the cadmium ratio with a thin $1/v$ detector. For $1/v$ epicadmium and thermal absorption, the probability of neutron capture is independent of velocity and is given by

$$-dN/N = dt/\tau_{th}$$

where τ_{th} is the mean thermal lifetime. This becomes after integration

$$N/N_0 = e^{-t/\tau_{th}} \tag{1}$$

Let $Q = N_0/\tau_{th}$ be the number of case histories of neutrons investigated during a time interval. At mean time, t_s, after source emission, a neutron is slowed to the thermal region. Because of $1/v$ epicadmium absorption, N_{th}/τ_{th} will be the reduced number of neutrons slowed below the cadmium cutoff energy. At $t = t_s$, $N/N_0 = N_{th}/N_0$ and Eq. (1) becomes

$$N_{th}/N_0 = e^{-t_s/\tau_{th}} \tag{2}$$

If a source of emission rate Q is present in the medium, $Q = N_0/\tau_{th}$ where N_0/τ_{th} is the neutron absorption rate in the medium. N_0 is the equilibrium number of neutrons present in the medium at any instant, and N_{th} and N_{Cd} are the thermal and epicadmium numbers, respectively. Equation (2) may be rewritten as

$$N_{Cd}/N_0 = 1 - e^{-t_s/\tau_{th}} \tag{3}$$

The activities with and without cadmium of the $1/v$ detector will be proportional to the corresponding neutron densities

$$\begin{aligned} A_{Cd}/A_0 &= 1 - e^{-t_s/\tau_{th}} \\ &= t_s/\tau_{th} \quad \text{for} \quad t_s/\tau_{th} \ll 1 \end{aligned} \tag{4}$$

Walker (*1*) has measured the ratio of epicadmium-to-bare capture, $\int_0^\infty A_{Cd} r^2\, dr / \int_0^\infty A_0 r^2\, dr$ with a small BF_3 detector

in a water solution at room temperature containing 0.1883 mole/liter H_3BO_3 and a Ra-Be source. He obtained a value of 0.0358 ± 0.0030 for the fraction of neutrons captured in the epicadmium region above 0.5 v.

For σ_B = 755b and σ_H = 0.332b, $v\Sigma_B$ = 18840 sec^{-1} and $v\Sigma_H$ = 4855 sec^{-1} for this solution. Then $\tau_{th} = 1/(v\Sigma_B + v\Sigma_H) = 42.2 \times 10^{-6}$ sec. As a result, the slowing-down time from Eq (4) is t_s = 1.54 ± 0.13 μsec. The atom density of boron is only 1/600 that of hydrogen in this solution, so the slowing-down time in pure water will be the same from 5 Mev to 0.5 v. Krieger (*2*) has calculated a value of 1.6 μsec for slowing down to 0.35 v in water at 293 °K. Haynam and Crouch (*3*) have obtained a value of 1.7 μsec for slowing down to 0.35 v and 1.1 μsec to 0.8 v with a Monte Carlo calculation. Effects due to chemical binding were neglected. Crouch's measured value of 5.2 μsec (*4*) to 0.35 v seems to have a systematic error. Walker's experiment would indicate little effect due to chemical binding.

This method of extracting a mean life is valid if the longest slowing down time is small compared to the mean thermal life so that the chance of capture is strictly proportional to the individual slowing down time, independent of its length. As t_s/τ_{th} becomes larger the calculated result obtained is smaller than the true value. For Walker's experiment, the bias may be of the order of 2% which is small compared with the experimental error.

Another quantity of interest may be obtained from measurements with a $1/v$ detector in any medium. Assume a small amount of $1/v$ poison is added to the medium but not enough to perturb the neutron distribution. The probability of capture of a neutron in the poison is proportional to the time the neutron spends in the medium,

$$-dN/N = v\Sigma_p\, dt = k dt.$$

If N neutrons are emitted by the source,

$$A_{Cd}/A_0 = \frac{\int_{i=1}^{N} N_i\, t_{ie}/N,}{\int_{i=1}^{N} N_i\, t_{i0}/N,} \qquad (5)$$

$$A_{Cd}/A_0 = \bar{l}_e/\bar{l}_0$$

where $\bar{l}_e$ is the mean epicadmium life of a neutron and $\bar{l}_0$ is the mean (total) life. For a $1/v$ medium $\bar{l}_0 = \tau_{th}$. Since epicadmium capture is included in the definition of $\bar{l}_e$, $t_s \geqq \bar{l}_e$.

REFERENCES

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3. G. E. Haynam and M. F. Crouch, *Nuclear Sci. and Eng.* **2**, 626–630 (1957).
4. M. F. Crouch, *Nuclear Sci. and Eng.* **2**, 631–639 (1957).

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Effect of Interference between Resonance and Potential Scattering on Resonance Absorption

In many recent calculations (*1*) of resonance integrals, use has been made of the narrow resonance (NR) and infinite mass (IM or NRIA) approximations, where the choice between the approximations has been based upon whether the practical width of the resonance is narrow or wide compared to the maximum energy loss of a neutron scattered from the resonance. For resonances having a practical width comparable to the maximum neutron energy loss, it has been recognized that the two approximations may disagree by a factor two or more and recent efforts have been made to obtain more accurate theories for such cases. Thus Chernick *et al.*, (*2*) have iterated the first order NR and IM approximations and Goldstein and Cohen (*3*), Bell and Bodine (*4*) have considered more general iterative and variational methods to connect the NR and IM limits.

In many of these theories either no account has been taken of the interference between resonance and potential scattering or it has been treated only very approximately. Recently, however, Rothenstein and Chernick (*5*) have shown that for absorption in a particular Bi lattice the interference is an important effect which increases absorption by about 40%. It is the purpose of this note to point out the general importance of interference for all resonances which are non-narrow (i.e., practical width $\gtrsim 4E/A$) and have appreciable scattering ($\Gamma_n \gtrsim \Gamma_a$). In general, for quantitative calculations of absorption in such resonances, the interference must be taken into account.

The effect on resonance absorption of interference between resonance and potential scattering can be seen most simply from numerical solutions of the integral equation for the neutron flux in an infinite homogeneous medium. The results of such calculations are given below for some U^{238} unbroadened Breit Wigner resonances and a Bi resonance with hydrogen as a moderator. In each case the resonance absorption probability was calculated both with and without interference and the results are compared.

Other calculations also show the importance of interference. Rothenstein (*6*) has evaluated the correction for narrow resonances and small interference. For fairly wide resonances and arbitrary interference, results may be obtained by iterative or variational techniques (*3*, *4*). For unbroadened Breit Wigner resonances, we have obtained analytic results with an approximate variational calculation. In these, the inclusion of interference increases the complexity of the calculation considerably and generalization to Doppler Broadened resonances may not be practical. Our variational results are compared with the numerical integration in Table I. The calculation used a flux trial function similar to that of Goldstein and Cohen (*3*), and a corresponding adjoint trial function. These were inserted into a Schwinger variational integral (*7*) for the neutron absorption and the resulting integrals evaluated. For the results reported here, the extremum was not found. Instead values of the parameters were chosen which simplified the result and were usually not far from the extremum. At any rate, errors in the absorption are of second order in the trial function errors.

Results for some strongly scattering U^{238} resonances in a 1:1 H/U mixture are summarized in Table I. For these calculations, Γ_γ = 0.025 ev, σ_p = 10 *b*, and σ_h = 20 *b*. It is