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LETTERS TO THE EDITORS

Space-Time Burnout of an Absorbing Slab

Consider a nonscattering absorbing slab extending from $x = 0$ to $x = D$. Assume a constant current of neutrons of strength ϕ_0 entering at $x = 0$ normal to the surface, and starting at time zero. Let the slab have a single absorbing species of microscopic cross section σ barns and initial number density N_0 nuclei per barn-cm. Then, at any time t and depth x, the neutron current and absorber number density satisfy the following simultaneous integral equations:

$$N(x, t) = N_0 \exp\left[-\sigma \int_0^t \phi(x, t')\, dt'\right] \tag{1}$$

$$\phi(x, t) = \phi_0 \exp\left[-\sigma \int_0^x N(x', t)\, dx'\right]. \tag{2}$$

These equations can easily be transformed to a pair of simultaneous differential equations by changing to a new set of variables:

$$u(x) = \sigma nvt \text{ at depth } x = \sigma \int_0^t \phi(x, t')\, dt' \tag{3}$$

$$v(t) = \text{depth in mean free paths} = \sigma \int_0^x N(x', t)\, dx'. \tag{4}$$

Equations (1) and (2) become

$$\partial v/\partial x = \sigma N_0 e^{-u} \tag{5}$$

$$\partial u/\partial t = \sigma \phi_0 e^{-v}. \tag{6}$$

A solution is easily found in the following form[1]

$$u = \ln\left[1 + e^{\sigma\phi_0 t - \sigma N_0 x} - e^{-\sigma N_0 x}\right] \tag{7}$$

$$v = \ln\left[1 + e^{\sigma N_0 x - \sigma\phi_0 t} - e^{-\sigma\phi_0 t}\right]. \tag{8}$$

The ϕ and N can be obtained by differentiation but often u and v are themselves more valuable. For example, $v(D)$ is the mean free path depth of the slab at any time.

A similar case of greater interest is simply to solve the same problem when the slab is subjected to a normally incident current from both sides. In this case let the total thickness of the slab be $2x$, and examine the conditions which apply at the center

$$\phi(x, t) = 2\phi_0 \exp\left[-\sigma \int_0^x N(x', t)\, dx'\right] \tag{9}$$

$$N(x, t) = N_0 \exp\left[-\sigma \int_0^t \phi(x, t')\, dt'\right]. \tag{10}$$

The solution proceeds in the same manner as above, yielding the following value for the slab depth (in mean free paths) as a function of time:

$$2v(x, t) = 2 \ln\left[1 + e^{N_0\sigma x - 2\phi_0\sigma t} - e^{-2\phi_0\sigma t}\right].$$

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[1] This solution proceeds from the fact that $\partial^2 u/(\partial x \partial t)$ and $\partial^2 v/(\partial x \partial t)$ are equal, so that $u + h(x) = v + g(t)$. The solution follows upon insertion into Eqs. (5) and (6) and use of boundary conditions.

Concerning the Theory of Control Sheets

In a recent paper (*1*), Wolfe derived a critical condition for a plane symmetric reactor with plane control sheets inserted, under the conditions that

$$\delta \gg \min(L, \sqrt{\tau}) \tag{1}$$

where δ is the spacing between sheets, and L, τ are the thermal diffusion length and age in the core material. In particular, a critical equation of the form

$$(\sin \mu\delta, \cos \mu\delta)(\alpha\lambda_1^N V_1 + \beta\lambda_2^N V_2) = 0 \tag{2}$$

has been given for N equally spaced sheets, where α, β, λ_1, λ_2 are functions of the material properties, and V_1, V_2 are vector functions of these properties.

Equation (2) was derived from the condition

$$(\sin \mu\delta, \cos \mu\delta) Q^N \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 0 \tag{3}$$

where

$$Q = \begin{pmatrix} \cos \mu\delta + R \sin \mu\delta & -\sin \mu\delta + R \cos \mu\delta \\ \sin \mu\delta & \cos \mu\delta \end{pmatrix} \tag{4}$$

and R is a function of material properties.

In the following we will show how the critical equation, Eq. (2), can be considerably simplified by working from Eqs. (3) and (4) in a somewhat different manner than was done in ref. *1*.

It was shown in ref. *1* that the eigenvectors V_1, V_2 of Q are

$$V_1 = \begin{pmatrix} S - RC \\ RS/2 - \sqrt{T^2 - 1} \end{pmatrix} \tag{5}$$