

neglected transient term. This criticism can be applied to the pulsed neutron problem which has also been treated by the energy dependent buckling concept.⁷

To give due credit to the idea of an energy dependent buckling, it must be admitted that it does constitute a practical, if unsophisticated, method of solving problems in finite geometry and generally yields results in reasonable accord with experiment. It is questionable, however, whether it provides a fundamental understanding of the basic physical processes involved and, moreover, coupled with diffusion theory it is unlikely to be of value in the region below the Bragg cutoff where the mean-free-path is of the same order as, or greater than, a characteristic transverse dimension. For example, the mean-free-path in graphite at the Bragg cutoff energy is about 20 cm.

Finally, it is the author's opinion that the only satisfactory method of dealing with problems of this type is through the transport equation analyzed in terms of its natural eigenfunctions. In this way, fundamental parameters such as κ and B are uniquely defined and can be obtained directly from experiment. Any other procedure, however effective it may be in reproducing the experimental results, can only be viewed as an artifice.

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⁷F. AHMED and A. K. GHATAK, *Nucl. Sci. Eng.*, **33**, 106 (1968).

Reply to "Comments on the Effect of Transverse Dimensions on the Diffusion Length in Crystalline Moderators"

In the preceding Letter Williams¹ has commented upon our recent paper, "Effect of Transverse Dimensions on the Diffusion Length of Neutrons in Crystalline Moderator Assemblies".² He criticizes our interpretation of his work and also our method of solving the problem. We would like to comment on these two aspects separately:

1. While studying diffusion of neutrons in an assembly with transverse buckling (B_1^2) less than the critical buckling (B_c^2), one is normally interested in the decay of the fundamental (or asymptotic) mode. According to Williams this asymptotic flux is space-energy separable. For example, a little beyond Eq. (5) of his letter he says "... the true solution which we know should be space-energy separable deep inside the medium (when K is unique)." (We do not agree with this and will comment on it a little later). Since we were *not* interested in transients, when we talked of neutron flux it was in relation to the fundamental model (or pseudo-asymptotic mode when B_1^2 was greater than B_c^2) and as such we did not misquote Williams.

It is true that in his paper, Williams³ starts with a very general form for the solution of the Boltzmann equation, yet his conclusions and final results have been deduced for

the asymptotic part of the flux, which, according to him, is space-energy separable. In his variational approach he also uses a trial function which is space-energy separable.

Williams certainly gives "an accurate" criterion for the critical transverse dimensions at which exponential decay ceases. However, when we stated that he does not explain the DeJuren and Swanson results,⁴ we were talking about the variation of K with buckling. We still maintain that he does not explain this explicitly.

2. We agree that the ansatz [Eq. (5) of Williams' Letter] that we have used is not an *exact* solution of the Boltzmann equation and we have said so in our paper. There, we have argued at some length that for small assemblies energy-dependent boundary conditions would be physically more appropriate. Once this is granted, our ansatz is a very good lowest order solution. We have shown that the terms we neglect are a few orders of magnitude smaller than those that we retain. (Along the axis of the assembly our solution is exact.) One frequently uses a similar approach in many branches of physics, quantum mechanics being one of them.

Our values of κ^2 (ν^2 in the notation of Williams) are buckling dependent and, in general, we cannot obtain infinite medium diffusion from this by subtracting some suitable buckling. What we have shown is that in the limit of $B_1^2 = 0$, our definition of κ^2 reduces to the standard definition. As such Williams remarks in the paragraph just above Eq. (5) are therefore inapplicable.

We are aware of the fact that the mean-free-path of cold neutrons is very large compared to that of neutrons in the thermal energy range. As mentioned in our paper, comparative studies of the solutions of the general transport equation, and of this equation under diffusion approximation, have been made by many workers (more recently by Nishina⁵). They generally find that diffusion theory gives results that are in close agreement with those obtained by transport theory, even in regions of parameters where diffusion theory would normally be expected to fail. Hence our use of the diffusion approximation is not unjustified.

We do not understand why Williams thinks that "the satisfactory method of dealing with *problems of this type* is through the transport equation analyzed in terms of its natural eigenfunctions." The problem that we pose—that of solving the Boltzmann equation in the diffusion approximation with energy dependent boundary conditions—is a clearly defined problem and, we feel, a physically more realistic one. Ours is the first attempt to solve it and we hope better solutions will soon be forthcoming.

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⁴J. A. DeJUREN and V. A. SWANSON, *J. Nucl. Energy*, **20**, 905 (1966).

⁵K. NISHINA, "Energy Dependent Diffusion Theory Treatment of Neutron Wave Propagation in a Finite Medium," PhD Thesis, The University of Michigan (1969).

¹M. M. R. WILLIAMS, *Nucl. Sci. Eng.*, **47**, 498 (1972).

²FEROZ AHMED, L. S. KOTHARI, and ASHOK KUMAR, *Nucl. Sci. Eng.*, **46**, 203 (1971).

³M. M. R. WILLIAMS, *Nukleonik*, **11**, 219 (1968).