VARIATIONAL METHODS

Title The Principles and Applications of Variational Methods

Author Martin Becker

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Reviewer Morton D. Kostin

Variational methods have been used in the nuclear-reactor field to derive approximate forms of the Boltzmann transport equation for neutrons, to estimate eigenvalues and weighted averages, and to obtain approximate solutions for the neutron flux. In his monograph Dr. Becker discusses least-squares variational functionals of the form

$$I = \int [H\phi - f]^2 \, p(x) \, dx, \qquad (1)$$

where $p(\chi)$ is an arbitrary positive function and $H\phi = f$ is the equation to be solved. Approximate solutions for ϕ are found by constructing a physically reasonable trial function, ϕ_t, which depends on a number of parameters, substituting ϕ_t into Eq. (1), and minimizing I to determine the values of the trial parameters. The outstanding feature of Dr. Becker's functional is that it can be used for nonlinear and non-self-adjoint problems.

Although the least-squares variational functional has several important advantages over other functionals, some questions regarding the accuracy of the approximate solutions remain. "The value of the functional", states Dr. Becker on p. 60, "can serve as a measure of the accuracy of the result." Does this mean that a trial function that produces a lower value of the functional than some other trial function will necessarily be a better approximation to the exact solution? For example, suppose that two different trial functions, ϕ_1 and ϕ_2, used to estimate the thermal flux in a certain region of a sub-critical reactor yield values of I_1 and I_2 for the least-squares variational functional. If $I_1 < I_2$, will ϕ_1 be a better representation of the flux in this region than ϕ_2? How can the errors of the approximate solution be estimated? A fuller discussion of the accuracy and limitations of the least-squares variational method would give the user of this method more confidence concerning the range of validity of his results.

A very readable but embarrassingly incomplete review of the basic principles of the variational and other trial-function methods is presented. The excellent work of Marshak, LeCaine, Schwinger, Francis, Stewart, Bohl, Krieger, Brooks, Calame, Devooght, Corngold, and many others who have made substantial contributions to this field is not even mentioned. Roussopoulos' article is noted, but no reference is made to the effort of Kahan and Rideau on which it is based. Moreover, it is not clearly stated that the main use of the variational method of Kahan, Rideau, and Roussopoulos is to calculate weighted averages of the neutron flux rather than to obtain approximate solutions for the neutron flux itself. Although Dr. Becker advances a superb case for his method, comparatively little attempt is made to balance his discussion by indicating to the reader the strong points of other techniques.

In addition to a clear analysis of many facets of the least-squares variational method such as boundary conditions, eigenvalues problems, and comparisions with other variational methods, the author presents illustrative numerical examples and a detailed application of his method to the problem of fuel depletion in a nuclear reactor, achieving a sound balance between theory and application. This well-written monograph can be recommended to applied mathematicians and physicists who are interested in new variational functionals and to engineers who are confronted with the extremely difficult problem of estimating neutron fluxes and other reactor quantities.

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BOOK ANNOUNCEMENTS

Although the following books will not be reviewed, they may be of interest to some of our readers:

Corrosion of Zirconium Alloys, American Society for Testing and Materials, Philadelphia, 1964, vi + 142 pp, cloth bound, 15 x 23 cm, $7.00

Assessment of Radioactivity in Man, International Atomic Energy Agency, Vienna, 1965, 17 x 25 cm, Vol. I: viii + 395 pp, $8.00; Vol. II: xii + 650 pp, $13.00

Medical Radioisotope Scanning, International Atomic Energy Agency, Vienna, 1964, 17 x 25 cm, Vol. I: xii + 559 pp, $11.50; Vol. II: xii + 470 pp, $9.50

Elementary Dynamics of Particles, H. W. Harkness, Academic Press, New York, 1964, ix + 219 pp, cloth or paper bound, 13 x 20 cm, $6.00 or $2.95

Physics and Material Problems of Reactor Control Rods, International Atomic Energy Agency, Vienna, 1964, 796 pp, cloth bound, 16 x 24 cm, $15.00

Advances in Hydroscience. Vol. I, Ven Te Chow, Academic Press, New York, 1964, x + 442 pp, cloth bound, 14.6 x 22.8 cm, $15.00