Letters to the Editors

Radial Buckling Modes Near the Source of an Exponential Column*

A set of data deleted from the published version (*1*) of the author's thesis (*2*) has proved quite valuable as an illustrative exercise for students of nuclear reactor engineering. The solution of the Helmholtz "wave" equation for a homogeneous cylindrical exponential column (*3*) with azimuthally independent source in the $z = 0$ plane is

$$\phi(r, z) = \sum_{k=1}^{\infty} A_{0k} J_0(\alpha_{0k} r) \sinh \gamma_{0k}(h - z),$$

where the buckling eigenvalues are $B^2{}_{0k} = \alpha^2{}_{0k} - \gamma^2{}_{0k}$ and the A_{0k} depend upon the source conditions.

TABLE I

OBSERVED RELATIVE FLUX DISTRIBUTION

Radius (in.)	Observed total foil activation (counts/min)
0	18.740×10^4
1	17.785×10^4
2	15.455×10^4
3	13.161×10^4
4	10.675×10^4
5	8.297×10^4
6	6.191×10^4
7	4.575×10^4
8	3.232×10^4
9	2.188×10^4
10	1.318×10^4

$$R(r) = 1.201^{\pm 0.025} \times 10^5 J_0\left(\frac{2.405}{2.405} \cdot 0.2111^{\pm 0.0035} r\right) + 4.62^{\pm 0.25} \times 10^4 J_0\left(\frac{5.520}{2.405} \cdot 0.2111^{\pm 0.0035} r\right) + 1.62^{\pm 0.16} \times 10^4 J_0\left(\frac{8.654}{2.405} \cdot 0.2111^{\pm 0.0035} r\right)$$

Clearly, near a source of the column as in Figure 3, of ref. *1*, the radial portion, $R(r)$, of the general solution should show the higher radial buckling modes, α_{0k}, $k = 1, 2, 3, \cdots$ The boundary condition that $R(\rho + d) = 0$ is satisfied by containing a series of J_0 Bessel functions such that the kth zero of the kth term is at $\rho + d$, where d is the extrapolation distance to be determined.

The radial flux distribution in a natural uranium column with $\rho = 10.5$ in., 4.1 in. from the source plane, was observed by foil activation techniques. The Los Alamos least-squares computer code (*4*) was used to separate the higher modes by fitting the function

$$R(r) = \sum_{k=1}^{n} b_k J_0(\alpha_{0k} r)$$

where the constraining boundary condition is met by

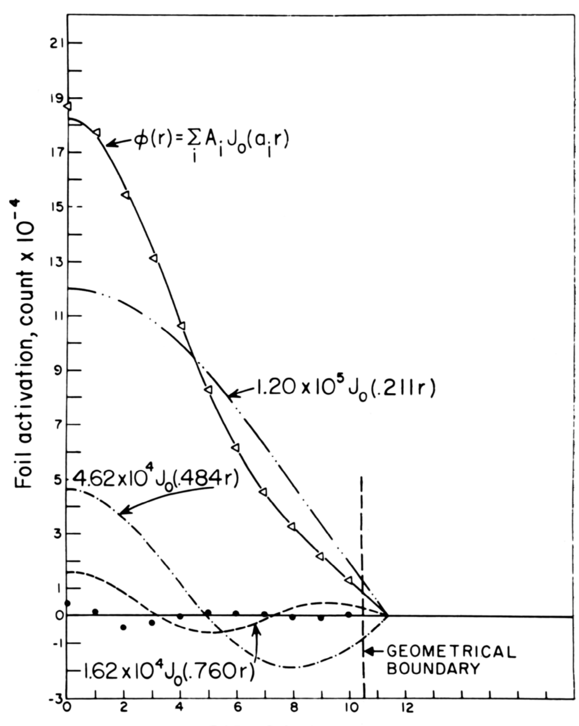

FIG. 1. Radial flux distribution, 4.1-in. level

setting

$$\alpha_{0k} = \frac{j_{0k}}{j_{01}} \alpha$$

leaving

$$\alpha = j_{01}/(\rho + d)$$

the only free parameter in the argument. The j_{0k} are the kth zeros of the J_0 Bessel function.

Table I lists the typical corrected observations of γ-counts/min from the foils (U^{235}, U^{238} or natural U) as well

* Work performed under the auspices of the U. S. Atomic Energy Commission.

as results of the least squares fit. Figure 1 graphically displays the three ($n = 3$) modes determined by the computer. Note that in Fig. 1 the residuals are plotted as dots leading to the suspicion of the existence of another mode. The student should be able to justify, on a statistical basis, the inability of the computer program to resolve a fourth mode.

REFERENCES

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