Table I also gives the measured values of $\phi(0)/\phi(r_0)$ from Ref. 1. For enrichments >20%, the measured centerline flux ratios from Ref. 1 are apparently in error since they fall considerably above the values for $\bar{\phi}/\phi(r_0)$. The flux depression factor calculated from Eqs. (2) and (3) shows the sensitivity to enrichment expected for enrichment changes up to 100%.

As noted in Ref. 1, the "method of successive generations" can be used to calculate the centerline flux ratio; however, it is difficult to apply in many cases because the necessary functions have been tabulated in Ref. 2 only for $0.5 \leqslant r_0\Sigma_t \leqslant 2.0$. For the rod sizes under consideration, hand calculations using this method are not feasible for enrichments >20%. A single calculation was made for the 0.953-cm-diam rod at 20% enrichment, resulting in a value of $\phi(0)/\phi(r_0) = 0.36$, as opposed to the measured value 0.57 given in Ref. 1.

An alternate method amenable to hand calculation of both the average and the centerline flux in rods has been derived by Bonalumi.[5] To determine the flux within the rod, Bonalumi takes the flux due to a unit cylindrical shell source as the sum of an asymptotic and transient component and then superimposes the contributions due to all such elementary cylindrical shells occupying the space outside the rod.

His result for the flux in a rod of radius $r_0$ is

$$\phi(r) = I_0(kr) + \lambda T(r_0,r) \ , \tag{4}$$

where $k$ is the positive root of the equation

$$\frac{k}{\Sigma_t} = \tanh \frac{k}{\Sigma_s} \ , \tag{5}$$

$$\lambda = \frac{Dk}{\beta K_1(kr_0)} \ , \tag{6}$$

$$D = \frac{\Sigma_a}{k^2} \ , \tag{7}$$

and $\beta$ is a coefficient <1, defined by

$$\beta = \frac{2\Sigma_a}{\Sigma_s} \frac{\Sigma_t^2 - k^2}{k^2 - \Sigma_t\Sigma_a} \ . \tag{8}$$

Both $k$ and $\beta$ are tabulated as functions of $\Sigma_s/\Sigma_t$ in Ref. 4. The function $T$ in Eq. (4) is a complicated integral involving products of Bessel functions that can be calculated in finite terms only for $r = 0$ and $r = r_0$. Closed form solutions are given in Ref. 5 for these two cases and for the average value. Results of calculations using Bonalumi's method are shown in Table I. The values calculated using Bonalumi's method are in good agreement with both the results of blackness theory and with the single calculation made using the method of successive generations.

Based on the calculations presented here, it appears that the measured flux ratios reported in Ref. 1 are too high for enrichments >5%, with the error increasing with increasing enrichment. One possible explanation for the anomalous results of Gibson and Anno is that there was a substantial epithermal component to their indium activations, which they failed to take into account. In any event, it has been demonstrated[5] that the flux shape in solid cylindrical rods shows large deviations from the diffusion theory shape even for natural uranium rods. It is not possible to determine a unique value of $k_{eff}$ that will normalize diffusion theory to measured results for use in engineering estimates. Attempts to do so will show that the $k_{eff}$ depends not only on enrichment but on rod diameter and on the measured quantity chosen for normalization. Bonalumi's method[5] is recommended for engineering estimates of the thermal-neutron flux shape in a rod.

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## REFERENCES

1. JAMES E. GIBSON and J. N. ANNO, "Thermal-Neutron Flux Depression in Cylindrical $UO_2$ Fuel Rods," *Nucl. Technol.*, **45**, 193 (1979).

2. G. W. STUART and R. W. WOODRUFF, "Method of Successive Generations," *Nucl. Sci. Eng.*, **3**, 339 (1958).

3. G. W. STUART, "Multiple Scattering of Neutrons," *Nucl. Sci. Eng.*, **2**, 617 (1957).

4. K. M. CASE, F. deHOFFMAN, and G. PLACZEK, *Introduction to the Theory of Neutron Diffusion*, Vol. I, U.S. Government Printing Office, Washington, D.C. (1953).

5. R. BONALUMI, "Theoretical Thermal Neutron Flux Shape in a Solid Cylindrical Rod," *Energ. Nucl.*, **8**, *1*, 71 (1961).

# REPLY TO "COMMENTS ON 'THERMAL-NEUTRON FLUX DEPRESSION IN CYLINDRICAL $UO_2$ FUEL RODS' "

To quote a reviewer of our Note,[1] "the flux depression problem is a difficult one analytically as well as experimentally." We are quite aware of the apparently anomalous experimental results at large enrichment mockups. Indeed, on *theoretical* grounds, the comments[2] on our results appear plausible. In our experiments, we experienced some difficulty in aligning and accurately locating the central axial indium wire. To minimize the possible effect of this problem, every experiment was performed twice and the larger measured flux depression was accepted. The "smoothness" of the curve drawn to the experimental data is evidence of small scatter in the data.

As pointed out in our Note, no correction was made for the perturbation caused by the central 0.51-mm (20-mil)-diam indium wire. This effect is certainly not completely negligible (especially at high-enrichment mockups) considering the diameter of the fuel rods (0.953 cm in most cases). Because of the blackness of the absorber at high-enrichment mockups, the presence of the indium wire would reduce the amount of flux depression from the actual situation, a trend that is in agreement with Sullivan's comments[2] on this Note.

As suggested by Sullivan, probably the most serious difficulty encountered in the experiments is the matter of accounting for epithermal-neutron activation of the indium wires. We failed to comment in the original Note that the measured cadmium ratio for indium (corrected for cadmium cover thickness effects) at the site of the experiment is 7.7.

Thus, the indium wires experienced epicadmium activation of

$$\text{epicadmium activation} = \frac{1}{CR - 1} = 0.14^{\underline{6}} \ .$$

We chose to assume that this relatively small fraction of epicadmium activation attenuated approximately as the thermal-neutron activation. Because of the complexity of the mockup mixtures (see Table I in the Note), which were selected to mock up the 2200 m/s absorption cross section of $UO_2$ at various enrichments, detailed account of the behavior of epithermal absorption was not attempted. One can only set an extreme upper bound to the possible error introduced by epithermal neutrons by assuming no attenuation in the absorbing material. For example, for the 0.953-cm-diam rod mocking up the 5% enrichment, the upper bound correction is to reduce the reported value of 0.86 to

$$\frac{0.86 - 0.15}{1.00 - 0.15} = 0.83^{\underline{5}} \ ,$$

or, hence, a 3% decrease from the reported measured value. This extreme correction would only reduce the 100% enrichment value (the worst case) from 0.43 to 0.33, still a large factor away from the <0.001 value suggested by Sullivan.[2]

While we will admit possible underestimates of the true thermal-neutron flux depression by our mockup experiments, we believe that our results are representative of the actual situation.

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#### REFERENCES

1. JAMES E. GIBSON and J. N. ANNO, "Thermal-Neutron Flux Depression in Cylindrical $UO_2$ Fuel Rods," *Nucl. Technol.*, **45**, 193 (1979).

2. RAYMOND P. SULLIVAN, "Comments on 'Thermal-Neutron Flux Depression in Cylindrical $UO_2$ Fuel Rods,'" *Nucl. Technol.*, **50**, 108 (1980).

## FURTHER COMMENTS ON "THERMAL-NEUTRON FLUX DEPRESSION IN CYLINDRICAL $UO_2$ FUEL RODS"

Reference 1 discusses two possible correction factors, both of which would tend to reduce the measured flux ratios reported in Ref. 2, in the direction indicated by the calculations presented in Ref. 3. The correction due to the perturbation caused by the central indium wire must be small since it occupies <1% of the rod volume even for the smallest rods measured (0.25 in. in diameter).

The correction due to epithermal-neutron activation was estimated in Ref. 1, using 7.7 as the measured cadmium ratio at the site of the experiment and assuming this value is appropriate for both low- and high-enrichment rods. A second assumption was that the epicadmium activation attenuates approximately as the thermal-neutron activation, which is equivalent to assuming that the cadmium ratios at the surface and at the center of the rods are the same.

Both of the above assumptions appear questionable for the range of measurements reported in Ref. 2. For rods with low absorption, the cadmium ratio at both the surface and the centerline is probably close to the measured value in the unperturbed flux. For highly absorbing rods, one would expect the cadmium ratio *even at the rod surface* to be considerably reduced due to two effects:

1. Spatial shielding of the indium wire surface facing the rod, which, as an upper limit for a flat wire surrounding a thermally black rod, would reduce the thermal activation by a factor of 2.

2. The depression of thermal flux in the moderator, which, for highly absorbing rods, is probably the most important effect.

As shown by the calculations made in Ref. 1, the correction to the measured centerline-to-edge thermal flux ratio is larger for highly absorbing rods and could become very large as the cadmium ratio approaches one.

To a first approximation, the assumption that the epicadmium activation attenuates approximately as the thermal-neutron activation is probably adequate for low enrichment rods when epicadmium neutrons account for a small fraction of the activations. For highly enriched rods, a much larger fraction of the activations will be from epithermal neutrons, and a measurement of the surface and centerline cadmium ratios in the presence of the experiment would appear to be necessary.

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#### REFERENCES

1. JAMES E. GIBSON and J. N. ANNO, "Reply to 'Comments on "Thermal-Neutron Flux Depression in Cylindrical $UO_2$ Fuel Rods,"'" *Nucl. Technol.*, **50**, 109 (1980).

2. JAMES E. GIBSON and J. N. ANNO, "Thermal-Neutron Flux Depression in Cylindrical $UO_2$ Fuel Rods," *Nucl. Technol.*, **45**, 193 (1979).

3. RAYMOND P. SULLIVAN, "Comments on 'Thermal-Neutron Flux Depression in Cylindrical $UO_2$ Fuel Rods,'" *Nucl. Technol.*, **50**, 108 (1980).