

Lessons

Topic

Unlocking Energy: Fission vs. Fusion

OBJECTIVES

Students will:

- Explore how isotope symbols can be used to determine the number of subatomic particles using an online interactive simulation.
- Model mass defect using particle-level representations.
- Calculate and compare the energy released in fusion and fission reactions.
- Investigate innovations in plasma technology.

Overview

Many people do not realize the massive amount of energy that is locked up in matter. In fact, the nucleus of any atom contains huge magnitudes of potential energy! Nuclear binding energy is the energy needed to disassemble the nucleus of an atom into its component parts. These component parts are neutrons and protons, which are collectively called nucleons.

In this activity, students will be introduced to the concepts of mass defect and nuclear binding energy. Then, using an online interactive simulation, students will review atomic mass units (amu) and explore how isotope symbols can be used to determine the number of subatomic particles. Using pictorial representations, students will model the protons and neutrons before and after formation of a nucleus. Students will explore the idea of nuclear binding energies in the context of fusion as they calculate the amount of energy released in the fusion reaction of deuterium and tritium using Einstein's published $E=mc^2$ equation. As a point of comparison, they will calculate energy released in the fission of U-235.

As extension options, students can create an infographic on the properties of plasma that includes at least one innovation in plasma technology, and they can control the Remote Glow Discharge Experiment (RGDX) that is located at the U.S. Department of Energy's Princeton Physics Plasma Laboratory.

Grade Band

9–12

Timing

Prep

Up to one hour to gather materials, prepare suggested videos

Activity

1 or 2 class periods, depending on class length and activities included.

Performance Expectation(s)

Students that understand the concept can:

- Develop models to illustrate the changes in the composition of the nucleus of the atom and the energy released during the process of fission, fusion, and radioactive decay (HS-PS1-8).
- Communicate scientific and technical information (e.g. about the process of development and the design and performance of a proposed process or system) in multiple formats (including orally, graphically, textually, and mathematically). (HS-PS2-6)

NGSS Standards

<p><u>Science & Engineering Practices:</u></p> <p>Developing and Using Models</p> <ul style="list-style-type: none"> • Develop a model based on evidence to illustrate the relationships between systems or between components of a system. <p>Using Mathematics and Computational Thinking</p> <ul style="list-style-type: none"> • Create and/or revise a computational model or simulation of a phenomenon, designed device, process, or system. • Use mathematical, computational, and/or algorithmic representations of phenomena or design solutions to describe and/or support claims and/or explanations. 	<p><u>Disciplinary Core Ideas:</u></p> <p>PS1.C: Nuclear Processes</p> <ul style="list-style-type: none"> • Nuclear processes, including fusion, fission, and radioactive decays of unstable nuclei, involve release or absorption of energy. • The total number of neutrons plus protons does not change in any nuclear process. 	<p><u>Crosscutting Concepts:</u></p> <p>Energy and Matter</p> <ul style="list-style-type: none"> • In nuclear processes, atoms are not conserved, but the total number of protons plus neutrons is conserved.
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Essential questions

- How are mass defect and binding energy related?
- How does the amount of energy released in a fusion reaction compare to the amount of energy released in a fission reaction?
- What are the properties and applications of plasma? Where can plasma be found (natural vs. human produced)?

Materials and Equipment:

- Computers or other devices connected to the internet
- Scientific or graphing calculator
- Copies of the *PhET SIMULATION QUESTIONS*, one per student

- Copies of the *Mass Defect and Binding Energy* video capture sheet, one per student
- Copies of the *YOU Be the Expert: Energy Released in Fusion and Fission Reactions* sheet, one per student

Prior Student Knowledge

Prior to beginning the lesson, students should be familiar with the following key vocabulary terms.

Key Vocabulary

(<https://www.nrc.gov/reading-rm/basic-ref/glossary/>)

Nucleons—Common name for a constituent particle of the atomic nucleus. At present, applied to protons and neutrons, but may include any other particles found to exist in the nucleus.

Isotopes—Two or more forms (or atomic configurations) of a given element that have identical atomic numbers (the same number of protons in their nuclei) and the same or very similar chemical properties but different atomic masses (different numbers of neutrons in their nuclei) and distinct physical properties. Thus, carbon-12, carbon-13, and carbon-14 are isotopes of the element carbon, and the numbers denote the approximate atomic masses. Among their distinct physical properties, some isotopes (known as radioisotopes) are radioactive because their nuclei emit radiation as they strive toward a more stable nuclear configuration. For example, carbon-12 and carbon-13 are stable, but carbon-14 is unstable and radioactive.

Nuclear binding energy—The minimum energy that would be required to disassemble the nucleus of an atom into its component parts.

Mass defect—The amount by which the mass of an atomic nucleus differs from the sum of the masses of its constituent particles, being the mass equivalent of the energy released in the formation of the nucleus.

Nuclear fusion—A reaction in which at least one heavier, more stable nucleus is produced from two lighter, less stable nuclei. Reactions of this type are responsible for enormous releases of energy, such as the energy given off by stars.

Nuclear fission—The splitting of the nucleus of an atom into nuclei of lighter atoms, accompanied by the release of energy.

Plasma—An ionized gas that allows both species, ions and electrons, to coexist and into which sufficient energy is provided to free electrons from atoms or molecules.

Procedure

1. Provide background information and review the lesson objectives.
2. As an activator, show the:

- Binding Energy Video: https://youtu.be/Xe_cZX-eZRc
(Show the following segments: 0:00 to 0:47 and 5:43 to 8:45.)

In their notebooks, students should answer these questions in their own words:

What is binding energy and where does it come from?

What holds an atom's nucleus together?

Using equitable calling strategies, ask students to volunteer their answers and record key points until you have a comprehensive “class” answer to each question.

3. Next, inform students that they will review atomic mass units (amu) and explore how isotope symbols can be used to determine the number of subatomic particles using an online interactive (PhET) simulation. Pass out copies of the *PhET*

Simulation Questions. Depending upon the number of available devices connected to the internet, have students work individually or in small groups to complete the handout. This handout can also be hosted on a collaborative platform.

- Then, tell students that they will use pictorial representations to model the protons and neutrons before and after formation of a nucleus. Hand out the Mass Defect and Binding Energy video capture sheet to students.

Show the video: Mass Defect/Binding Energy Models and Calculations:

<https://www.youtube.com/watch?v=qYUTigPx6A>

Instruct students to complete the capture sheet as they watch.

(From start to min 2:27, students will complete Part I. At this point, pause the video and debrief using equitable calling strategies to make sure all students captured the information on their sheet.)

- Using what they learned in the PhET simulation and the videos, students will complete Part II of the video capture sheet. (Note: Teachers should watch the Mass Defect/Binding Energy video in its entirety before teaching the lesson, as it contains hints for how to complete Part II, including step-by-instructions for completing the calculations.) After students complete Part II, you may want to show the remainder of the video to the class so that students can check their work.
- Finally, split the class into Fusion Experts on one side of the room and Fission Experts on the other side. Hand out the *YOU Be the Expert: Energy Released in Fusion and Fission Reactions* sheet. Instruct students to complete the reaction calculations for the side of the sheet that corresponds to their assigned group (Fusion Expert or Fission Expert). When finished, students should go to the “Source” link below the reaction to watch a video to check their calculations:

<https://tinyurl.com/tvxnth>

Then, have students partner up with someone from the opposite group to share their calculations with each other and compare the difference in magnitude between the fusion and fission reactions. In their notebooks, have students respond to the following short prompt:

*As you can see, both physical processes produce massive amounts of energy from atoms. In general, which process releases more energy **per reaction**: fusion or fission? Support your answer with evidence from your calculations. Would you expect the same to hold true **per unit of nuclear mass**? Explain.*

- To wrap up the lesson, debrief to emphasize the following key points:
 - Fission and fusion are both physical processes that produce enormous quantities of energy from atoms.
 - Generally, an individual fission releases more energy **per reaction** than an individual fusion.
 - A typical fission releases about 200 MeV (e.g. U-235 fissions at 202.5 MeV).
 - An individual fusion releases around 10–20 MeV (e.g. deuterium-tritium fusion releasing around 17 MeV).
 - Per unit weight**, fusion releases more energy. Fusion reactions are themselves millions of times more energetic than chemical reactions. The fission of heavy elements is highly exothermic which releases about 200 million eV compared to burning coal which only gives a few eV. The amount of energy released during nuclear fission is millions of times more efficient per mass than that of coal considering only 0.1 percent of the original nuclei is converted to energy.
 - Chemical reactions involve changes in the electron orbits which are small compared to the changes in the relative positions of the electrical charge of the nuclear particles in the nucleus of the atom.

You can use this infographic to help summarize fission vs fusion:

<https://www.energy.gov/ne/downloads/infographic-fission-vs-fusion-whats-difference>

Supplemental information on conservation laws in nuclear reactions may be found here:

<https://www.nuclear-power.net/laws-of-conservation/law-of-conservation-of-energy/>

Extend the learning options

1. Plasma is the fourth state of matter, a very hot ionized gas that makes up the Sun and most of the universe. Students will create an infographic on the properties of plasma that should include at least one innovation in plasma technology using the following resources:

- physics.anu.edu.au/research/themes/plasma.php
- www.aa.washington.edu/research/ZaP/research/plasmaOverview
- <https://www.britannica.com/science/plasma-state-of-matter>

2. Students will experiment with the Remote Glow Discharge Experiment (RGDX) found in the U.S. Department of Energy's Princeton Physics Plasma Laboratory: www.pppl.gov/RGDX

They can control the gas pressure inside the tube, the voltage produced by the power supply that makes the plasma, and the strength of an electromagnet surrounding the plasma.

To start using the RGDX, press the "Access RGDX" button on the left side of the screen or click on the following link: scied-web.pppl.gov/rgdx/

Questions students should consider as they experiment include:

- What is plasma?
- Why does the plasma glow?
- What is pressure and how do you control it?
- What is the electrode voltage and how do you control it?
- What are electromagnets and how do you control them?
- How does a magnetic field interact with the plasma?

PHET SIMULATION QUESTIONS—ISOTOPES AND ATOMIC MASS

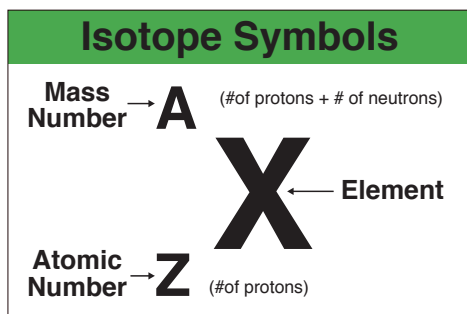
Directions: Open the Isotopes and Atomic Mass simulation (tinyurl.com/phetsimulation1) Experiment with the “Make Isotopes” tab of the simulation.

Use the simulation to answer the following essential questions:

- How do you add and remove neutrons?
- What does adding or subtracting neutrons do to the atomic symbol and molecular model?
- What does abundance in nature mean?
- Which is heavier, a proton or a neutron?
- What makes an atom stable or unstable?

Once you feel comfortable using the simulation, answer the following questions:

1. What particles determine the mass number?
2. One isotope of carbon (C) has the same mass number and atomic mass since it was used as the standard definition of the atomic mass unit (amu). Which isotope is it and what is its atomic mass?
3. Why is mass number always a whole number?
4. Knowing that the element Lithium is atomic number 3, how many neutrons would each of the following isotopes of Lithium have? Also write a nuclear symbol for each following the format below:



<https://tinyurl.com/ybe9kj4g>

• **Lithium-5**

Nuclear Symbol:

Number of neutrons: _____

• **Lithium-6**

Nuclear Symbol:

Number of neutrons: _____

• **Lithium-7**

Nuclear Symbol:

Number of neutrons: _____

• **Lithium-8**

Nuclear Symbol:

Number of neutrons: _____

Mass Defect and Binding Energy

Part I

Define

Mass Defect: _____

Binding Energy: _____

Pictorial Representations

Mass (Before)	Mass (After)

Follow-Up Questions:

1. Compare the masses before and after formation of a nucleus. Which one is bigger?

2. Which isotope is represented in this example?

3. What is the mass defect converted into?

Part II: Calculating Average Binding Energy Per Nucleon for an Iron-56 Nucleus

1. Isotope Symbol for Iron-56

Given Values (in amu):

- $m_p = 1.007825 \text{ u}$
- $m_n = 1.008665 \text{ u}$

2. What is the total mass of the nucleons (all the protons and all the neutrons) before the formation of the nucleus?
Show the calculation below. Report answer to six places after the decimal.

3. The tabulated value for the mass after the formation of an iron nucleus is 55.934939 u. Subtract the mass after the formation of the nucleus from the total mass of the nucleons before the formation of the nucleus to obtain the mass defect. Show your calculation for the mass defect below. Report answer to six places after the decimal.

4. Now you are ready to convert this mass into energy (in mega electron volts) using Einstein's famous mass-energy equivalence relationship, $E = mc^2$. Show your conversions/calculations in the box below making sure your units cancel properly:

Useful Information:

$$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$$

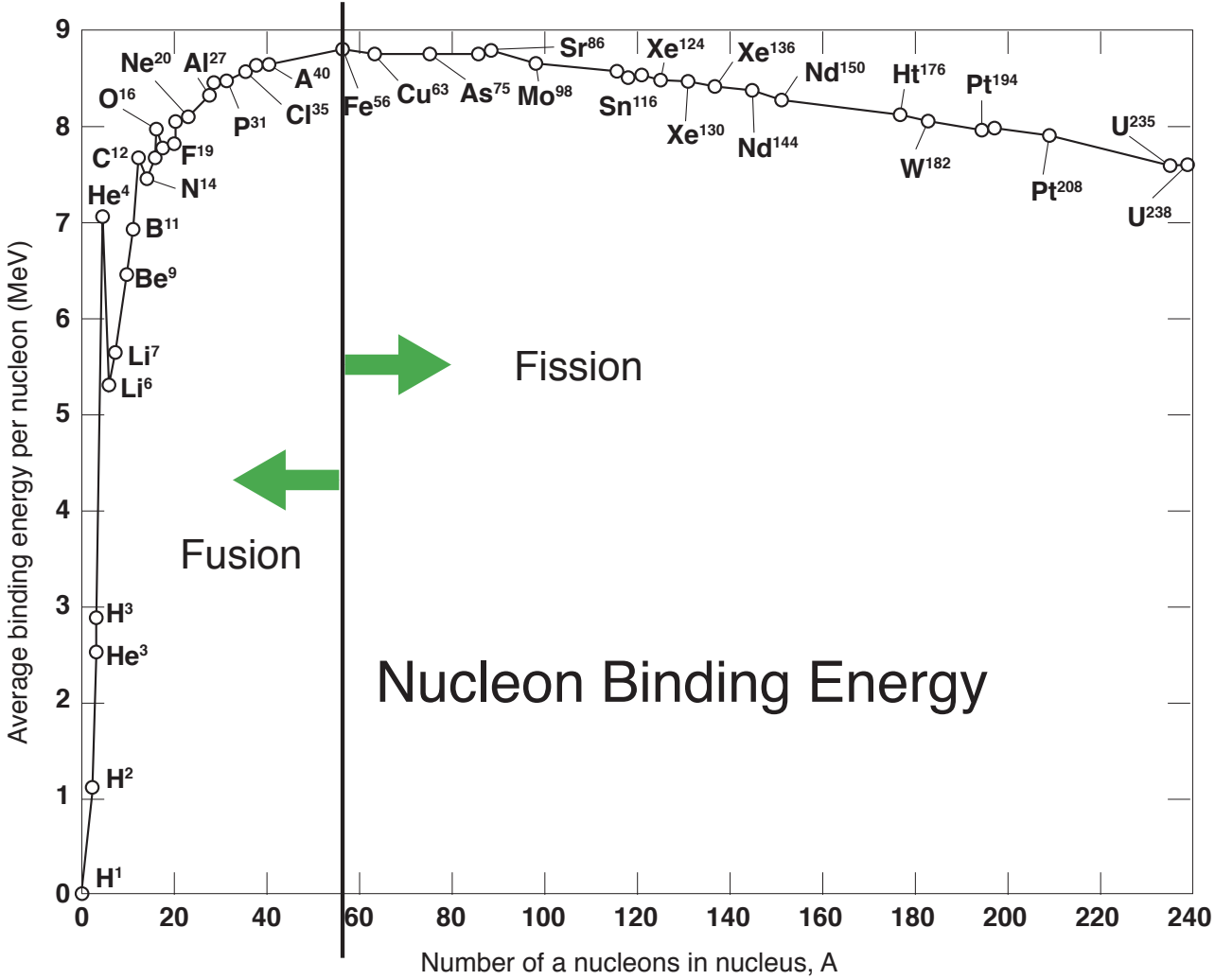
$$c = \text{speed of light} = 3.00 \times 10^8 \text{ m/s}$$

$$1 \text{ electron volt (eV)} = 1.60 \times 10^{-19} \text{ J}$$

$$1 \text{ mega electron volt (MeV)} = 10^6 \text{ eV}$$

5. To find the average binding energy per nucleon, divide the energy from #4 by the total number of nucleons in the iron nucleus. Show your calculation in the box below.

6. Evaluate the reasonableness of your answer by comparing it to the tabulated value for Fe-56 in the chart below. Is your answer reasonably close to the value shown in the chart?

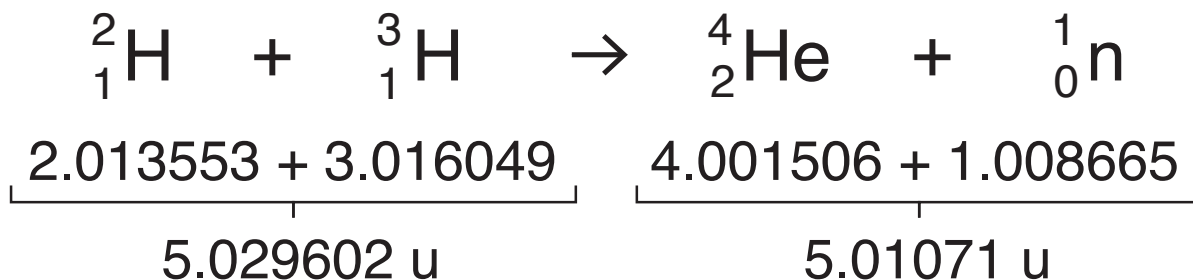


http://www.mpoweruk.com/images/binding_energy.gif

YOU Be the Expert: Energy Released in Fusion and Fission Reactions

Energy Released in Nuclear Fusion Reaction (Do this side if you're a fusion expert!)

Reaction—Deuterium and tritium react to form helium and a neutron as shown in the following nuclear equation:



Source: <https://tinyurl.com/tvxnth>

1. Calculate the mass defect in amu and convert to kg. Hint: $1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$

2. Next, calculate the energy associated with this reaction in kilojoules (kJ). Hints: $E = mc^2$ where $c = 3.00 \times 10^8 \text{ m/s}$, $1,000 \text{ J} = 1 \text{ kJ}$, and $6.01 \times 10^{23} \text{ particles} = 1 \text{ mol}$. Multiply your final answer by Avogadro's number (6.02×10^{23}) to get the energy released for one mole of helium nuclei.

PHET SIMULATION QUESTIONS—ISOTOPES AND ATOMIC MASS

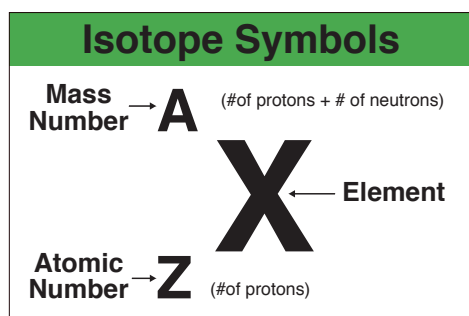
Directions: Open the Isotopes and Atomic Mass simulation (tinyurl.com/phetsimulation1) Experiment with the “Make Isotopes” tab of the simulation.

Use the simulation to answer the following essential questions:

- How do you add and remove neutrons?
Click and drag the small grey spheres from the grey container into the blue isotope sphere to add neutrons. Click and drag the small grey spheres out of the blue isotope sphere to remove neutrons.
- What does adding or subtracting neutrons do to the atomic symbol and molecular model?
When adding neutrons, the mass number of the isotope symbol increases by one for each neutron added and the nuclear model includes an extra neutron.
When removing neutrons, the mass number of the isotope symbol decreases by one for each neutron that is removed, and the nuclear model loses a neutron.
- What does abundance in nature mean?
When adding neutrons, the mass number of the isotope symbol increases by one for each neutron added and the nuclear model includes an extra neutron.
When removing neutrons, the mass number of the isotope symbol decreases by one for each neutron that is removed, and the nuclear model loses a neutron.
- Which is heavier, a proton or a neutron?
A neutron is a little heavier than a proton. (Note: The mass of a proton is 1.00728 amu and the mass of a neutron is 1.00867 amu.)
- What makes an atom stable or unstable?
An atom is stable if the forces among the particles that make up the nucleus are balanced. An atom is unstable (radioactive) if these forces are unbalanced; if the nucleus has an excess of internal energy. Instability of an atom’s nucleus may result from an excess of either neutrons or protons. (Source: <https://tinyurl.com/y789z3jw>)

Once you feel comfortable using the simulation, answer the following questions:

1. What particles determine the mass number?
The atomic mass number is determined by the sum of all protons and neutrons in the nucleus.
2. One isotope of carbon (C) has the same mass number and atomic mass since it was used as the standard definition of the atomic mass unit (amu). Which isotope is it and what is its atomic mass?
The isotope is carbon-12 and its atomic mass is 12 (the sum of 6 protons and 6 neutrons).
3. Why is mass number always a whole number?
Mass number is always a whole number because it represents the number of protons plus the number of neutrons of an isotope and you never have part of a proton or neutron. On the other hand, atomic mass is a weighted average (it takes into account the percentage of each isotope), and it is not a whole number.
4. Knowing that the element Lithium is atomic number 3, how many neutrons would each of the following isotopes of Lithium have? Also write a nuclear symbol for each following the format below:



<https://tinyurl.com/ybe9kj4g>

• Lithium-5

Nuclear Symbol:

Number of neutrons: 2

• Lithium-6

Nuclear Symbol:

Number of neutrons: 3

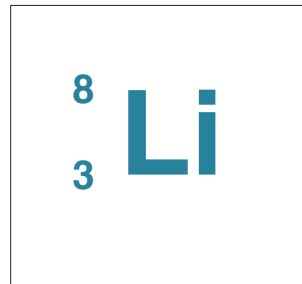
• Lithium-7

Nuclear Symbol:

Number of neutrons: 4

• Lithium-8

Nuclear Symbol:

Number of neutrons: 5

Mass Defect and Binding Energy

Part I

Define

The amount by which the mass of an atomic nucleus differs from the sum of the masses of its

Mass Defect: _____

constituent particles, being the mass equivalent of the energy released in the formation of the nucleus.

Binding Energy: _____

Pictorial Representations

Mass (Before)	Mass (After)
M_B 	M_A ${}^7_4\text{Be}$

Source: <https://www.youtube.com/watch?v=qYUTigPx6A>

Follow-Up Questions:

1. Compare the masses before and after formation of a nucleus. Which one is bigger?

Mass before the formation of a nucleus is bigger than the mass after the formation of the nucleus ($m_b > m_a$).

2. Which isotope is represented in this example?

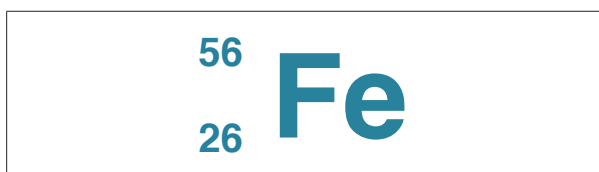
The beryllium-7 isotope is represented in this example.

3. What is the mass defect converted into?

Binding energy

Part II: Calculating Average Binding Energy Per Nucleon for an Iron-56 Nucleus

1. Isotope Symbol for Iron-56



Given Values (in amu):

- $m_p = 1.007825 \text{ u}$
- $m_n = 1.008665 \text{ u}$

2. What is the total mass of the nucleons (all the protons and all the neutrons) before the formation of the nucleus?
Show the calculation below. Report answer to six places after the decimal.

There are 26 protons and 30 neutrons in Fe-56:

$$m_p = 1.007825\text{u} \times 26 = 26.203450 \text{ u}$$

$$m_n = 1.008665\text{u} \times 30 = 30.259950 \text{ u}$$

$$m_{\text{before}}(\text{total}) = 56.463400 \text{ u}$$

3. The tabulated value for the mass after the formation of an iron nucleus is 55.934939 u. Subtract the mass after the formation of the nucleus from the total mass of the nucleons before the formation of the nucleus to obtain the mass defect. Show your calculation for the mass defect below. Report answer to six places after the decimal.

$$\text{Mass defect} = m_{\text{before}} - m_{\text{after}}$$

$$\text{Mass defect} = 56.463400\text{u} - 55.934939\text{ u} = 0.528461\text{ u}$$

4. Now you are ready to convert this mass into energy (in mega electron volts) using Einstein's famous mass-energy equivalence relationship, $E = mc^2$. Show your conversions/calculations in the box below making sure your units cancel properly:

Useful Information:

$$1\text{ u} = 1.66 \times 10^{-27}\text{ kg}$$

$$c = \text{speed of light} = 3.00 \times 10^8\text{ m/s}$$

$$1\text{ electron volt (eV)} = 1.60 \times 10^{-19}\text{ J}$$

$$1\text{ mega electron volt (MeV)} = 10^6\text{ eV}$$

First, we need to convert the mass from u \rightarrow kg

$$0.528461\text{ u} \times (1.66 \times 10^{-27}\text{ kg} / 1\text{ u}) = 8.77 \times 10^{-28}\text{ kg}$$

Next, calculate the energy equivalent using Einstein's mass-energy equivalence relationship,

$$E = mc^2 = 8.77 \times 10^{-28}\text{ kg} (3.0 \times 10^8\text{ m/s})^2 = 7.89 \times 10^{-11}\text{ Joules}$$

*Note: 1 Joule = 1 kg*m²/s²*

Then, convert the energy from Joules to electron volts (eV) and finally to mega electron volts (MeV) as follows:

$$E = 7.89 \times 10^{-11}\text{ J} \times (1\text{ eV} / 1.60 \times 10^{-19}\text{ J}) = 4.93 \times 10^8\text{ eV} \times (1\text{ MeV} / 10^6\text{ eV}) = 493\text{ MeV}$$

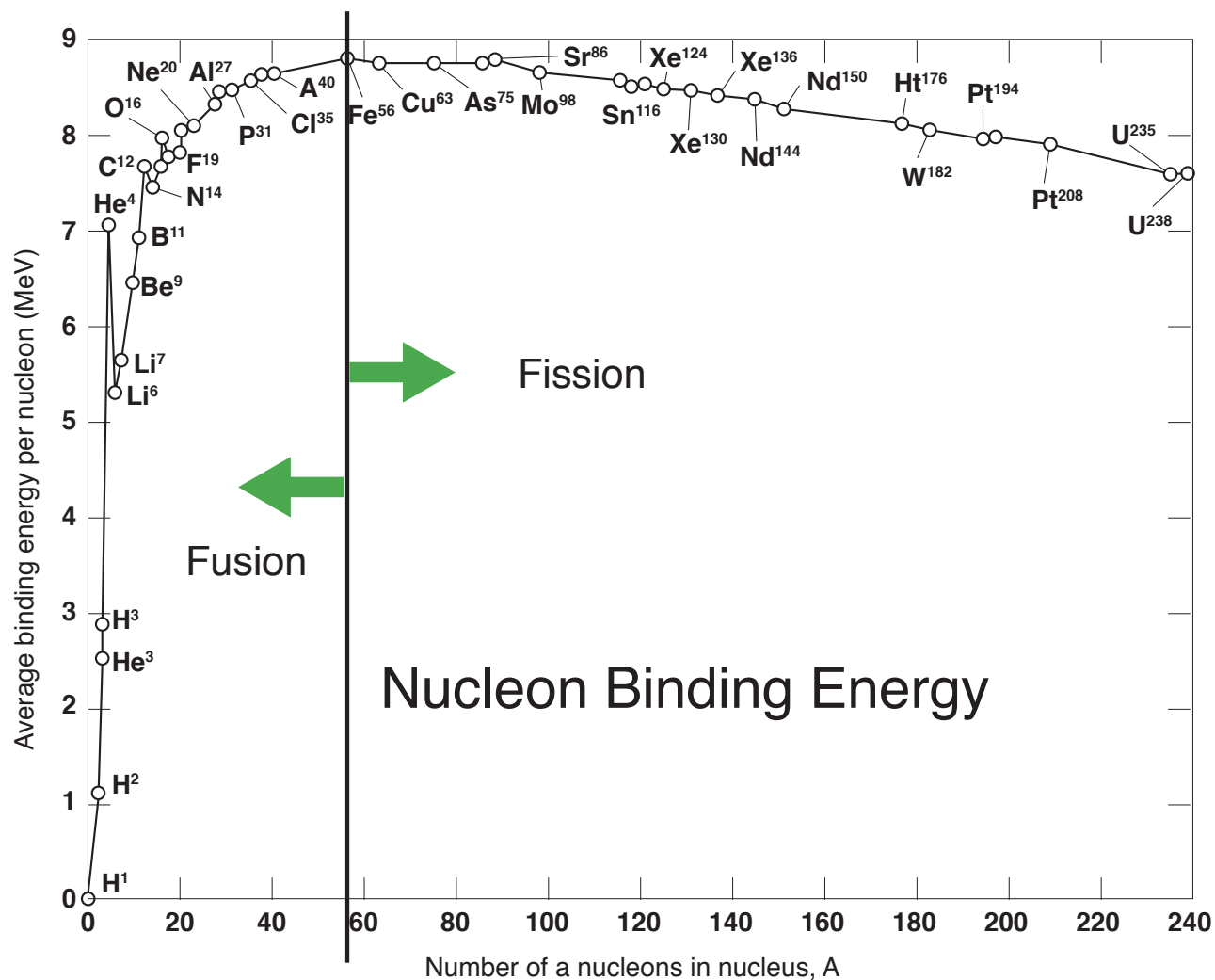
5. To find the average binding energy per nucleon, divide the energy from #4 by the total number of nucleons in the iron nucleus. Show your calculation in the box below.

Since Fe-56 has 56 nucleons, divide 493 MeV by 56 to get the average binding energy per nucleon:

$$493\text{ MeV} / 56 = 8.812\text{ MeV}$$

6. Evaluate the reasonableness of your answer by comparing it to the tabulated value for Fe-56 in the chart below. Is your answer reasonably close to the value shown in the chart?

Yes, as can be seen in the chart below (vertical line at 56 nucleons), the tabulated value is higher than 8.5 MeV and just under the 9 MeV. Therefore, the calculated value is reasonable.

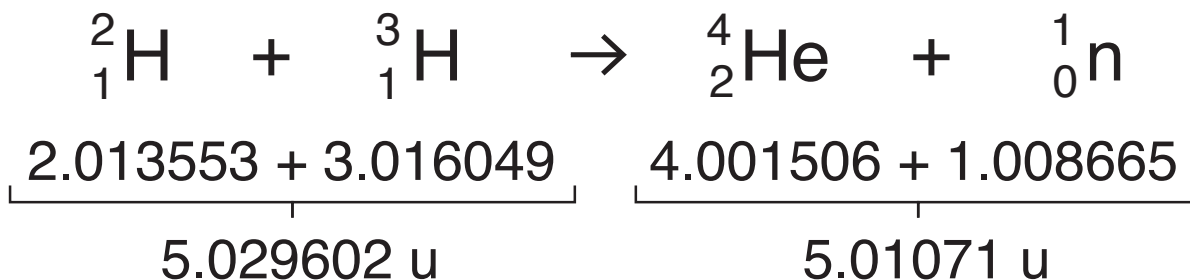


http://www.mpoweruk.com/images/binding_energy.gif

YOU Be the Expert: Energy Released in Fusion and Fission Reactions

Energy Released in Nuclear Fusion Reaction (Do this side if you're a fusion expert!)

Reaction—Deuterium and tritium react to form helium and a neutron as shown in the following nuclear equation:



Source: <https://tinyurl.com/tvxnth>

1. Calculate the mass defect in amu and convert to kg. Hint: 1 u = 1.66 x 10⁻²⁷ kg

The masses are given in unified atomic mass units (u).

$$\text{Mass defect} = m_{\text{before}} - m_{\text{after}} = 5.029602 \text{ u} - 5.01071 \text{ u} = 0.019431 \text{ u}$$

$$0.019431 \text{ u} \times (1.66 \times 10^{-27} \text{ kg} / 1 \text{ u}) = 3.23 \times 10^{-29} \text{ kg}$$

2. Next, calculate the energy associated with this reaction in kilojoules (kJ). Hints: E = mc² where c = 3.00 x 10⁸ m/s, 1,000 J = 1 kJ, and 6.01 x 10²³ particles = 1 mol. Multiply your final answer by Avogadro's number (6.02 x 10²³) to get the energy released for one mole of helium nuclei.

$$E = mc^2 = 3.23 \times 10^{-29} \text{ kg} \times (3.00 \times 10^8 \text{ m/s})^2 = 2.91 \times 10^{-12} \text{ J}$$

Next, convert joules to kilojoules:

$$2.91 \times 10^{-12} \text{ J} \times (1 \text{ kJ} / 1000 \text{ J}) = 2.91 \times 10^{-15} \text{ kJ}$$

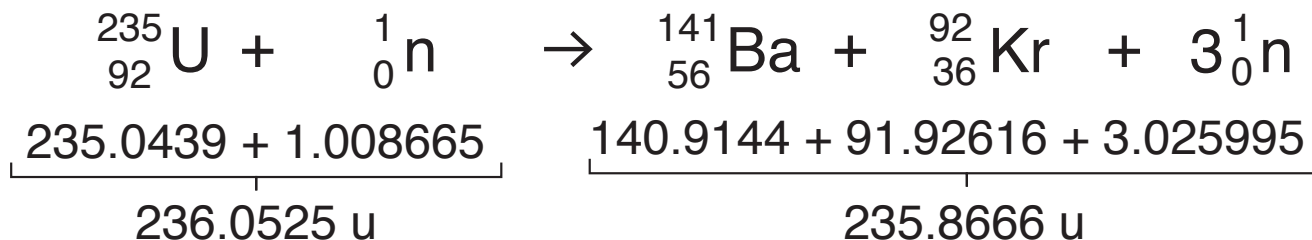
For one mole of helium nuclei, multiply by Avogadro's number (6.02 x 10²³):

$$(2.91 \times 10^{-15} \text{ kJ}) (6.02 \times 10^{23} / 1 \text{ mol}) = 1.75 \times 10^9 \text{ kJ} / \text{mol helium nuclei}$$

YOU Be the Expert: Energy Released in Fusion and Fission Reactions

Energy Released in Nuclear Fusion Reaction (Do this side if you're a fission expert!)

Reaction—Nuclear fission of uranium-235 to yield barium-141, krypton-92 and three neutrons as shown below:



Source: <https://tinyurl.com/tvxnth>

1. Calculate the mass defect in amu and convert to kg. Hint: 1 u = 1.66 x 10⁻²⁷ kg

The masses are given in unified atomic mass units (u).

$$\text{Mass defect} = m_{\text{before}} - m_{\text{after}} = 236.0525 \text{ u} - 235.8666 \text{ u} = 0.1859 \text{ u}$$

$$0.1859 \text{ u} \times (1.66 \times 10^{-27} \text{ kg} / 1 \text{ u}) = 3.09 \times 10^{-28} \text{ kg}$$

2. Next, calculate the energy released in the fission reaction in kilojoules (kJ). Hints: E = mc² where c = 3.00 x 10⁸ m/s, 1,000 J = 1 kJ, and 6.01 x 10²³ particles = 1 mol. Multiply your final answer by Avogadro's number (6.02 x 10²³) to get the energy released for one mole of U-235.

$$E = mc^2 = 3.09 \times 10^{-28} \text{ kg} \times (3.00 \times 10^8 \text{ m/s})^2 = 2.78 \times 10^{-11} \text{ J}$$

Next, convert joules to kilojoules:

$$2.78 \times 10^{-11} \text{ J} \times (1 \text{ kJ} / 1000 \text{ J}) = 2.78 \times 10^{-14} \text{ kJ}$$

For one mole of U-235, multiply by Avogadro's number (6.02 x 10²³):

$$2.78 \times 10^{-14} \text{ kJ} \times (6.02 \times 10^{23} / 1 \text{ mol}) = 1.67 \times 10^{10} \text{ kJ/mol U-235 nuclei}$$