value μ_1 is prevailing, it is justified to introduce in (9a) the first mode $p_1(E)$ of the adjoint thermalization operator. Observing relation (11) we get

$$\Sigma_{ij}^{(i)} = \mu_1^{(i)} . \tag{12}$$

Eq. (12) corresponds to Leslie's result[3]. The results of Selengut[1] and Lindenmeier[2] are obtained by a linear approximation to $p(E)$ satisfying Eq. (8),

$$p(E) \sim (2\, T_i - E). \tag{13}$$

This approximation guarantees both the balance of neutrons and of energy. Moreover the two approaches will agree for higher mass values since (13) is exactly the first mode of the adjoint heavy-gas operator.

Expression (9a) becomes now

$$\Sigma_{ij}^{(i)} = \frac{\int dE \int dE'\,(E'-E)\ \Sigma_s^{(i)}\ (E\rightarrow E')\ M(E,T_j)}{2(T_i - T_j)} \tag{14}$$

which can be evaluated analytically for the free gas scattering kernel (mass ratio A, free atom macroscopic scattering cross section Σ_0) yielding

$$\Sigma_{ij}^{(i)} = \frac{2A}{(A+1)^2}\ \Sigma_0^{(i)}\ \left(1 + \frac{T_i}{AT_j}\right)^{1/2} . \tag{15}$$

Pearce[4] has obtained the same formula (15) from a gas-kinetic point of view.

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Diffusion Coefficient of Thermal Neutrons for a Maxwellian Gas

The experimental values of the spectra and diffusion parameters of thermal neutrons are usually compared with those calculated on the basis of the free-gas model and on the basis of more realistic models, which account for the effects of the chemical binding (for example, see Ref. 1). In this way, it is possible to have a quantitative insight into the effect of the chemical binding on these values.

Following this line we obtain here an analytical expression for the diffusion coefficient of thermal neutrons for a gas of free protons that are in motion with a Maxwellian distribution of velocities. The diffusion coefficient for nonabsorbing media is defined in general as

$$D(v) = \frac{1}{3N\,\langle \sigma_s(v)\rangle\ [1 - \mu(v)]} , \tag{1}$$

where N is the atomic density of the considered medium and $\langle \sigma_s(v)\rangle$ and $\mu(v)$ are the microscopic scattering cross section and the average cosine of the scattering angle in the laboratory system (l.s.), respectively, which are functions of the neutron velocity, v.

As far as the scattering cross section is concerned, we use in Eq. (1) the analytical expression of $\langle \sigma_s(v)\rangle$ given in Refs. 2 and 3.

For the average cosine of the scattering angle in the l.s., $\mu(v)$, we proceed as follows. Under the hypothesis that the scattering cross section $\sigma_0(v_r)$ is independent of the relative velocity v_r of the two colliding particles[3] and the scattering is isotropic in the center of mass system, we may define

$$\mu(v) = \langle \cos\eta(v)\rangle_{\text{eff}} \equiv \frac{\langle \sigma_s(v)\cos\eta(v)\rangle}{\langle \sigma_s(v)\rangle} , \tag{2}$$

where

$$\begin{aligned} &\langle \sigma_s(v)\cos\eta(v)\rangle \\ &= \frac{\sigma_0}{v}\int_0^{2\pi}\frac{d\phi}{2\pi}\int_{-1}^{+1} d\left(\frac{\cos\beta}{2}\right)\int_{-1}^{+1} d\left(\frac{\cos\alpha}{2}\right) \\ &\quad \times \int_0^{\infty} v_r \cos\eta\, M_{T_0,M}\, dV. \end{aligned} \tag{3}$$

The notations used in Eq. (3) are those indicated in Ref. 4, pp 101 and 397. Performing the integration appearing on the right-hand side of Eq. (3), one obtains[5] for a free proton gas

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$$\mu(x)\frac{\sigma_s(x)}{\sigma_0} = \frac{2}{x^2} + \frac{10}{3\sqrt{\pi}}\frac{1}{x^3} + \frac{2}{3\sqrt{\pi}}\frac{e^{-x^2}}{x}\left(1 - \frac{5}{x^2}\right) + 2\left(\frac{1}{3} - \frac{2}{x^2}\right)\mathrm{Erf}(x) + \sqrt{\pi}\left(1 - \frac{1}{x^2}\right)\frac{e^{x^2}}{x}\mathrm{Erf}(x)\,[1 - \mathrm{Erf}(x)]\,, \qquad (4)$$

where $x = v/v_0$ and $v_0 = \sqrt{2kT}$.

Substituting Eq. (4) into Eq. (1) and with the appriate expression for $\langle\sigma_s(v)\rangle$ yields

$$ND(x) = \left\{3\sigma_0\left[\frac{1}{3}\frac{e^{-x^2}}{x\sqrt{\pi}} - \frac{2}{x^2} + \left(\frac{1}{3} + \frac{9}{2x^2}\right)\mathrm{Erf}(x) - \sqrt{\pi}\left(1 - \frac{1}{x^2}\right)\frac{e^{x^2}}{x}\mathrm{Erf}(x)\{1 - \mathrm{Erf}(x)\} - \frac{10}{3\sqrt{\pi}}\frac{1}{x^3}(1 - e^{-x^2})\right]\right\}^{-1} = \{3\sigma_0 S(x)\}^{-1}. \qquad (5)$$

In Fig. 1 we plot the values of $\mu(x)$ and $ND(x)$ calculated from Eqs. (4) and (5), respectively, assuming $\sigma_0 = 20.7$ b.

The Maxwellian average of the diffusion coefficient, defined as

$$N\langle D\rangle = \frac{\int_0^\infty \frac{vM_{T_0,M}(v)}{3\sigma_s(v)[1 - \mu(v)]}dv}{\int_0^\infty vM_{T_0,M}(v)dv}\,, \qquad (6)$$

is obtained from measurements of the diffusion properties of thermal neutrons.

From Eqs. (5) and (6) the expression for $N\langle D\rangle$ of a free proton gas is

$$N\langle D\rangle = \frac{2}{3\sigma_0}\int_0^\infty \frac{x^3 e^{-x^2}}{S(x)}dx = \frac{2}{3\sigma_0}G. \qquad (7)$$

Numerical calculations yield 0.656 as the value of G.

We calculated the values of $\langle D\rangle$ from Eq. (7) for H_2O, C_6H_6 and $C_{12}H_{10}$ at $T = 300°K$. From the comparison with the values calculated by theoretical models[6,7,10], which account for the chemical binding, and with experimental results[6,8,9,10,11], we can conclude that using the free-proton gas model overestimates the diffusion coefficient for H_2O and hydrocarbons by a factor of 2 to 3. We neglect oxygen and carbon contributions.

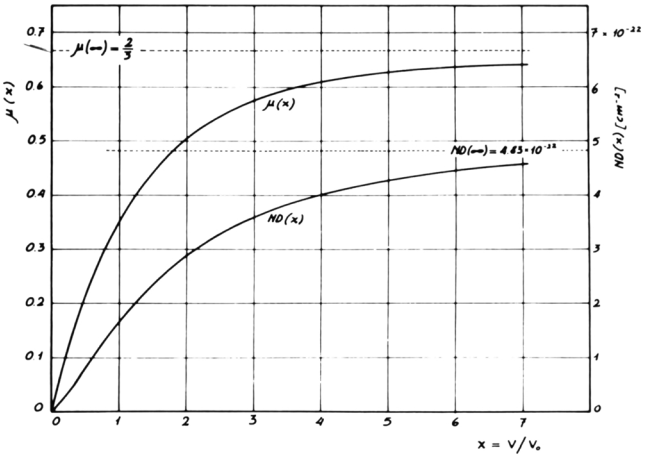

Fig. 1. Diffusion coefficient and average cosine of the scattering angle of neutrons for a free proton gas.

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An Approximate Solution of the Two-Overlapping-Thermal-Group Diffusion Equations*

The use of two overlapping thermal groups to represent the spatially dependent thermal-neutron spectrum has been considered by several investigators[1-3]. The purpose of this letter is to describe

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