of nuclide abundance or level parameters, resonance I dominates the shape of the flux near the resonant energy. Then a simple estimate of the capture rate in resonance II may be obtained by integrating the capture cross section from resonance II times the flux determined by the presence of resonance $I$ in the absence of resonance II.

The procedure is to express the capture cross section in resonance II in terms of the line shape of resonance I and a modifying function. We use unbroadened line shapes and expand $\sigma_{a_{2}}$ in the difference $x_{1}{ }^{2}-x_{2}{ }^{2}$, where $x_{i}=2\left(E-E_{r_{i}}\right) / \Gamma_{i} ; E_{r_{i}}$ is the resonance energy and $\Gamma_{i}$ is the total width.

For the flux we use the intermediate representation of Goldstein and Cohen ${ }^{3}$ and introduce their parameter $\beta_{\lambda}$ given by

$$
\beta_{\lambda}^{2}=1+\frac{\sigma_{0}}{s+\lambda \sigma_{p}} \frac{\Gamma_{\gamma}+\lambda \Gamma_{n}}{\Gamma}
$$

where
the resonance parameters are those of the dominant resonance (I)
$\sigma_{0}$ is the peak cross section of resonance $I$,
$s$ is the effective moderator scattering per atom of type I
$\sigma_{p}$ is the total potential scattering per atom of type I
$\lambda$ is the intermediate representation parameter.

Performing the integration by a contour inte$\operatorname{gral}^{\mathrm{a}}$

$$
R I_{2}=\frac{\Gamma_{1}}{2 E_{r_{1}}} \int_{-\infty}^{\infty} \psi_{1} \sigma_{a_{2}} d x_{1}
$$

where $R I_{2}$ is the resonance integral for the second resonance

$$
\psi_{1} \approx\left(1+x_{1}^{2}\right) /\left(\beta_{\lambda}+x_{1}^{2}\right), \sigma_{a_{2}}=\frac{\Gamma_{\gamma 2}}{\Gamma_{2}} \frac{\sigma_{0,2}}{1+x_{2}^{2}}
$$

we find

$$
\begin{aligned}
R I_{2}= & I_{0,2} \frac{E_{2}}{E_{1}} \times \\
& \times\left[1+\frac{\left(1-\beta_{\lambda}^{2}\right) \Gamma_{1}}{\beta_{\lambda}} \frac{\Gamma_{2}+\beta_{\lambda} \Gamma_{1}}{\left(\Gamma_{2}+\beta_{\lambda} \Gamma_{1}\right)^{2}+4\left(E_{2}-E_{1}\right)^{2}}\right],
\end{aligned}
$$

where $I_{0,2}$ is the infinite-dilution resonance integral.

[^0]Consider as an example a mixture of $\mathrm{H}: \mathrm{U}^{238}=1$ and estimate the depression in the response of a gold foil from the interference between the 6.68 eV resonance of $\mathrm{U}^{238}$ and the 4.91 eV resonance in gold. We have for resonance I:

$$
\begin{aligned}
& E_{1}=6.68 \mathrm{eV} \\
& \sigma_{0}=2.192 \times 10^{4} \mathrm{barns} \\
& \Gamma_{1}=0.0264 \mathrm{eV} \\
& \sigma_{s}=20 \mathrm{barns} \\
& \lambda=0 \\
& \beta_{\lambda}^{2}=954 .
\end{aligned}
$$

For resonance II:

$$
\begin{aligned}
& E_{2}=4.91 \mathrm{eV} \\
& \Gamma_{2}=0.1406 \mathrm{eV}
\end{aligned}
$$

By substitution we find $\frac{R I_{2}}{I_{0,2}}=0.69$; the largest part of the ratio, 0.734 , comes from the term $E_{2} / E_{1}$. It may be fairly argued that the ratio $E_{2} / E_{1}$ should be omitted, since it comes from neglecting the $1 / E$ dependence of the flux. In fact, if we put $\beta_{\lambda}=1$ (no resonance flux depression) we find $R I_{2} / I_{0,2}$ given by: $R I_{2} / I_{0,2}=E_{2} / E_{1}$. We therefore suggest modifying the formula given above by dropping the ratio $E_{2} / E_{1}$. When this is done for the example cited, $R I_{2} / I_{0,2}=0.94$. An exact calculation ${ }^{1}$ gives a value of 0.92 for this ratio.

Charles N. Kelber

## Argonne National Laboratory <br> Argonne, Illinois 60440

Received October 29, 1964
Revised December 8, 1964

## Resonance Integrals for $\mathbf{U}^{233}$ Fission and $\mathrm{Th}^{232}$ Capture

The resonance integrals for $\mathrm{U}^{233}$ fission and $\mathrm{Th}^{232}$ capture have been measured relative to the resonance capture integral of $\mathrm{Au}^{197}$ by means of the cadmium-ratio-activation technique ${ }^{1}$. Dilute detector foils were irradiated in an 11.5 cm diam water hole at the center of the TRX critical facility ${ }^{1}$. The TRX is a water-moderated lattice of cylindrical, slightly enriched uranium metal and $\mathrm{UO}_{2}$ fuel rods. The epithermal flux spectrum in the water hole was approximately proportional to $1 / E$ except for the flux peak above 25 keV .

Figure 1 shows the disc-shaped cadmium box.

[^1]

Fig. 1. Cadmium box.

The $\gamma$ activities of the foils were measured by means of a NaI scintillation counter. Details of the foils and $\gamma$ counting are summarized in Table I.

The neutron flux spectrum was calculated in $P_{1}$ approximation assuming one-dimensional, cylindrical geometry by means of the P3MG program ${ }^{2}$. This calculation employed one thermal-energy group and 54 groups from 0.625 eV to 10 MeV . The thermal-flux spectrum was obtained in 40 groups up to 1.4 eV by means of a space-independent calculation ${ }^{3}$, assuming the existence of a pure buckling. This was normalized to the one-group thermal flux from the P3MG calculation up to 0.625 eV . The spectrum is shown in Fig. 2.

The cadmium cutoff energy $\epsilon^{j}$ for each detector

[^2]was calculated, assuming infinite slab geometry, from the expression ${ }^{1}$
\[

$$
\begin{equation*}
\int_{\epsilon}^{\mathrm{sev}} \phi(E) \sigma^{j}(E) d E=\int_{0}^{\mathrm{sev}} \phi(E) \sigma^{j}(E) E_{2}(X) d E, \tag{1}
\end{equation*}
$$

\]

where

$$
X=\sum_{a}^{\mathrm{Cd}}(E) t
$$

The resonance integral $I^{j}$ above the cutoff energy was obtained from the relation (terms are defined in Table II)

$$
\begin{equation*}
I^{j}\left(\epsilon^{j}\right)=I^{A \mathrm{u}} \frac{(\mathrm{C} \mathrm{R}-1)^{\mathrm{Au}}}{(\mathrm{C} \mathrm{R}-1)^{j}} \frac{\sigma_{0}^{j}}{\sigma_{0}^{\mathrm{Au}}} \frac{F^{j}}{F^{A \mathrm{u}}} \tag{2}
\end{equation*}
$$

Calculated corrections were applied where necessary to account for departure from $1 / v$ dependence of the detector subcadmium cross sections and for slight foil self-shielding in this range. Corrections were also applied for self-shielding of the epicadmium detector activations and for departure of the epithermal flux spectrum from the $1 / E$ shape. For $U^{233}$, a correction to account for edge effects in the cadmium box was obtained by comparing the activations of the ring and center portions of the foil (see Fig. 1). These corrections are incorporated in the $F$ factors of Eq. (2). The results, as well as details of the calculations, are spelled out in Table II.

Natural activities of $\mathrm{U}^{233}$ and $\mathrm{Th}^{232}$ were subtracted. Perturbation of the results by aluminum or nickel activities was found to be negligible. Half-lives of $\mathrm{Au}^{198}$ and $\mathrm{Pa}^{233}$ were consistent with accepted values within the statistical accuracy of the counts. All uncertainties are compounded as though they were random. The $U^{233}$ result includes $1 \%$ to account for uncertainty of the cutoff energy.

The value for $\mathrm{Th}^{232}$ is in agreement with that obtained by Brose ${ }^{4}$ using the cadmium-ratio technique. His result of $82.7 \pm 1.8$ barn when put on the same basis as the present value (namely $I^{A u}=$ 1555 barn and $\sigma_{0}^{\text {Th }}=7.33$ barn) becomes $80.8 \pm 1.8$

[^3]TABLE I
Details of Detectors and $\gamma$ Counting

|  | $\mathrm{U}^{233}$ | $\mathrm{Au}^{197}$ | $\mathrm{Th}^{232}$ |
| :--- | :--- | :--- | :--- |
| Foil Composition | $2.8 \mathrm{wt} \% \mathrm{U}^{233}-\mathrm{Al}$ | $0.25 \mathrm{wt} \% \mathrm{Au}-\mathrm{Ni}$ | $2.9 \mathrm{wt} \% \mathrm{Th}-\mathrm{Al}$ |
| Foil Diameter (in.) | 0.488 | 0.387 | 0.387 |
| Foil Thickness (in.) | 0.006 | 0.002 | 0.004 |
| Activity Counted | Fission Products | $\mathrm{Au}^{198}$ | $\mathrm{~Pa}^{233}$ |
| Half-life (day) | --- | 2.7 | 27.4 |
| Window | 400 keV int. | $360-460 \mathrm{keV}$ | 50 keV int. |



Fig. 2. Flux spectrum.

TABLE II
Experimental Results

| Detector, $j$ | $\mathrm{Au}^{197}$ | $\mathrm{Th}^{232}$ | $\mathrm{U}^{233}$ |
| :---: | :---: | :---: | :---: |
| Cadmium thickness, $t$ (in.) <br> Measured cadmium ratio, C R <br> Calculated cadmium cutoff energy, $\epsilon^{j}(\mathrm{eV})$ | $\begin{aligned} & \hline 0.0205 \\ & 3.918(0.5 \%)^{b} \\ & (0.5)^{\mathrm{c}} \end{aligned}$ | $\begin{aligned} & 0.0205 \\ & 5.105(0.6 \%) \\ & (0.5)^{c} \end{aligned}$ | 0.0215 0.0320 <br> $31.20(0.5 \%)$ $32.11(0.5 \%)$ <br> 0.62 0.75 |
| Cross section at $0.0253 \mathrm{eV}, \sigma_{0}^{j}$ (barn) | 98.8 (0.3\%) | 7.33 (1.6\%) | 526 (0.6\%) |
| Calculated subcadmium non- $1 / v$ correction, $\Delta_{1}$ <br> Calculated subcadmium self-shielding correction, $\Delta_{2}$ <br> Calculated epicadmium self-shielding correction, $\Delta_{3}$ <br> Calculated correction to $1 / E$ epicadmium spectrum, $\Delta_{1}$ <br> Measured correction to infinite slab geometry for cadmium box, $\Delta_{5}$ <br> Total correction factor, $F^{j}=1+\Delta_{1} \ldots+\Delta_{\mathrm{s}}$ | $\begin{aligned} & +0.006 \\ & 0 \\ & +0.014 \\ & 0 \\ & 0 \\ & 1.020(1 \%) \end{aligned}$ | $\begin{aligned} & -0.005 \\ & 0 \\ & +0.010 \\ & +0.021 \\ & 0 \\ & 1.026 \text { (1.3\%) } \end{aligned}$ | -0.004 -0.006 0 +0.005 -0.010 $0.985(0.8 \%)$ |
| $\begin{aligned} & \text { Resonance integral above } \epsilon^{j}, I^{j} \text { (barn) } \\ & \text { Calculated correction to } \epsilon^{j}=0.5 \mathrm{eV} \text { (barn) } \end{aligned}$ | 1555 (2.6\%) ${ }^{\text {a }}$ | 82.5 | 771 748 <br> +27 +49 |
| $\%$ uncertainty assigned (excluding $\sigma_{0}{ }^{\prime}$ s and $I^{\text {Au }}$ uncertainties) <br> $\%$ uncertainty assigned (including $\sigma_{0}$ ' $s$ and $I^{\mathrm{Au}}$ uncertainties) <br> Resonance integral above 0.5 eV (barn) |  | $\begin{gathered} 2.0 \% \\ 3.6 \% \\ 82.5 \pm 3.0 \end{gathered}$ | $\begin{gathered} 1.8 \% \\ 3.3 \% \\ 798 \pm 26 \end{gathered}$ |

[^4]barn, compared with $82.5 \pm 1.7$ barn. It is not known how close to $1 / E$ was the spectrum in which the measurements of Brose were made. Johnston, et al. ${ }^{5}$, Tiren and Jenkins ${ }^{6}$ and Sampson ${ }^{7}$ have obtained values in the vicinity of 83 barn by activation techniques.

[^5]The situation with regard to $\mathrm{U}^{233}$ is less clear. Halperin, et al. ${ }^{8}$ obtained $875 \pm 40$ barn above 0.5 eV by radiochemical separation and counting of selected fission product activities. Bigham ${ }^{9}$ gets $739 \pm 36$ barn by counting fission product activity, whereas the present experiment yields $798 \pm 26$ barn. The reasons for these discrepancies are not apparent.

Integration of differential $\mathrm{U}^{233}$ fission cross section data has yielded results of 789 barn ${ }^{10}$, 775 barn $^{11}$ and 772 barn $^{12}$, all consistent with the present value. In the case of $\mathrm{Th}^{232}$, however, some adjustment of the resonance parameters is necessary in order to obtain a value in the vicinity of 83 barn $^{13,14}$.
J. Hardy, Jr.

Bettis Atomic Power Laboratory*
Pittsburgh, Pennsylvania
Received December 3, 1964
*Operated for the USAEC by Westinghouse Electric Corporation.


[^0]:    ${ }^{\text {a }}$ Suggested by E. Pennington, ANL. The contour lies in the upper half plane.
    ${ }^{3}$ R. GOLDSTEIN and E. R. COHEN, "Theory of Resonance Absorption of Neutrons," Nucl. Sci. Eng., 13, 132140 (1962).

[^1]:    ${ }^{1}$ J. HARDY, Jr, , D. KLEIN and G. G. SMITH, Nucl. Sci. Eng., 9, 341-345 (1961).

[^2]:    ${ }^{2}$ H. BOHL, Jr., et al., "P3MG-1-A One-Dimensional Multigroup P-3 Program for the Philco-2000 Computer,' WAPD-TM-272, Bettis Atomic Power Laboratory (1963).
    ${ }^{3}$ W. W. CLENDENIN, "MAGMA-A Philco 2000 Program for the Calculation of Scattering Kernels, Neutron Spectra and Few-Group Parameters for Thermal Neutrons,' WAPD-TM-373, Bettis Atomic Power Laboratory (1964).

[^3]:    ${ }^{4}$ M. BROSE, Thesis, Technische Hochschule Karlsruhe (1962).

[^4]:    ${ }^{\text {a }}$ Resonance integral standard ${ }^{14,}{ }^{15}$.
    ${ }^{\mathrm{b}}$ Errors quoted for cadmium ratios are $\%$ standard deviations obtained from the spreads of the individual measurements.
    ${ }^{c}$ Nominal values.

[^5]:    ${ }^{5}$ F. J. JOHNSTON, J. HALPERIN and R. W. STOUGHTON, J. Nucl. Energy, Part A: Reactor Sci., 11, 95-100 (1960).
    ${ }^{6}$ L. I. TIREN and J. M. JENKINS, "Resonance Shielding in Thorium-Graphite Mixtures,' AEEW-R-163, Winfrith (1962).
    ${ }^{7}$ J. B. SAMPSON, "Analysis of Activation Measurements of $\mathrm{Th}^{232}$ Resonance Captures in the Peach Bottom (40MW(E) Prototype HTGR) Critical Assembly,' GA-3069, General Atomic (1962).
    ${ }^{8}$ J. HALPERIN et al., Nucl. Sci. Eng., 16,245-247 (1963).
    ${ }^{9} \mathrm{C}$. B. BIGHAM, "Fission Resonance Integrals of $\mathrm{U}^{233}$, $\mathrm{U}^{235}, \mathrm{Pu}^{239}$ and $\mathrm{Pu}^{241}, ’$ CRRP-1183, Chalk River (1964).
    ${ }^{10}$ L. B. FREEMAN, Private communication.
    ${ }^{\text {"I }}$ S. TERASAWA, "The Effect of Epithermal Fission on Aqueous Homogeneous Reactors,' ORNL-2553, Oak Ridge (1958).
    ${ }^{12}$ J. CHERNICK and S. O. MOORE, Nucl. Sci. Eng., 6, 537-544 (1959).
    ${ }^{13}$ L. W. NORDHEIM, GA-3973 (1963).
    ${ }^{14}$ JAKOB WEITMAN, Nucl. Sci. Eng., 18, 246-259 (1964).
    ${ }^{15} \mathrm{~K}$. JIRLOW and E. JOHANSSON, J. Nucl. Energy, Part A: Reactor Sci., 11, 101-107 (1960).

