LETTERS TO THE EDITOR

COMMENTS ON "THERMAL-NEUTRON FLUX DEPRESSION IN CYLINDRICAL UO_2 FUEL RODS"

Gibson and Anno[1] presented results of flux measurements made on mockups of cylindrical fuel rods of UO_2 with varying enrichment. The measured center-to-edge thermal flux ratio, $\phi(0)/\phi(r_0)$, was given for three different rod diameters and various enrichments. For each measurement a k_{eff} value was determined that would force agreement with the measurements when used in the formula derived from diffusion theory, namely,

$$\frac{\phi(0)}{\phi(r_0)} = \frac{1}{I_0(kr_0)} \quad . \tag{1}$$

The suggestion was made that use of the experimentally determined k_{eff} values would permit use of the diffusion theory expression for engineering estimates of the thermal-neutron flux depression in cylindrical UO_2 fuel rods.

Although diffusion theory is known to underestimate the thermal flux depression in absorbing rods, the results of Gibson and Anno indicate that this holds true in UO_2 rods only for enrichment up to 20% and that at higher enrichments diffusion theory overestimates the flux depression. There is no physical explanation for such an anomalous result. Furthermore, the Gibson and Anno results show the rod centerline flux to be insensitive to enrichment above the 50% enrichment level. Even at 100% enrichment, their measured centerline fluxes remain >40% of the surface flux for rod diameters <0.5 in. For highly absorbing rods, one would expect the centerline thermal flux to decrease continuously and approach zero as Σ_a increases. Since Σ_a approximately doubles in going from 50 to 100% enrichment, one would expect the measured centerline fluxes to show a much larger sensitivity to enrichment changes in this range.

That the results in Ref. 1 are indeed in error can be demonstrated by calculation of the integral quantity $\bar{\phi}/\phi(r_0)$, where $\bar{\phi}$ is the average flux in the rod. The theoretical basis for calculation of this quantity is well established. Since $\phi(0) < \bar{\phi}$, it follows that $\bar{\phi}/\phi(r_0)$ must be greater than $\phi(0)/\phi(r_0)$. For highly absorbing rods, the ratio $\bar{\phi}/\phi(r_0)$ can be calculated using a formula derived[2] from blackness theory:

$$\frac{\phi(r_0)}{\bar{\phi}} = r_0 \Sigma_a \frac{(2-\beta)}{\beta} \quad , \tag{2}$$

where β is the rod blackness. The value β has been calculated as a function of $\chi = r_0\Sigma_t$ and the ratio of scattering to total cross section in the rod, $c = \Sigma_s/\Sigma_t$.

Values of β for $0.5 < \chi < 2.0$ are plotted in Fig. 8 of Ref. 3. For small values of χ or small values of c, an excellent approximation to β is given by[3]

$$\beta = \frac{(1-c)\,2r_0\Sigma_t[1 - P_c(r_0\Sigma_t)]}{1 - cP_c(r_0\Sigma_t)} \quad , \tag{3}$$

where P_c is the first collision probability for cylinders as tabulated in Ref. 4.

Calculations were made using thermal cross sections as defined in Ref. 1. Results for the rod sizes and enrichments studied in Ref. 1 are given in Table I. For comparison,

TABLE I

Flux Depression Factor for UO_2 Rods

Rod Diameter (cm)	Enrichment (%)	Flux Depression Factors			
		$\bar{\phi}/\phi(r_0)$		$\phi(0)/\phi(r_0)$	
		Eqs. (2) and (3)	Ref. 5	Ref. 5	Ref. 1
0.953	5	0.89	0.88	0.86	0.86
0.953	10	0.78	0.78	0.61	0.73
0.953	15	0.68	0.67	0.47	0.64
0.953	20	0.59	0.57	0.33[a]	0.57
0.953	50	0.30	0.29	0.04	0.47
0.953	75	0.21	0.20	0.001	0.45
0.953	100	0.15	[b]	[b]	0.43
0.635	15	0.79	0.78	0.63	0.81
0.635	100	0.23	0.23	0.01	0.50
1.27	15	0.58	0.57	0.33	0.53
1.27	100	0.11	[b]	[b]	0.40

[a]For this case a calculation of $\phi(0)/\phi(r_0)$ using the method of Ref. 2 gave a value of 0.36.

[b]Case was outside the range for which the required functions are tabulated.

Table I also gives the measured values of $\phi(0)/\phi(r_0)$ from Ref. 1. For enrichments >20%, the measured centerline flux ratios from Ref. 1 are apparently in error since they fall considerably above the values for $\bar{\phi}/\phi(r_0)$. The flux depression factor calculated from Eqs. (2) and (3) shows the sensitivity to enrichment expected for enrichment changes up to 100%.

As noted in Ref. 1, the "method of successive generations" can be used to calculate the centerline flux ratio; however, it is difficult to apply in many cases because the necessary functions have been tabulated in Ref. 2 only for $0.5 \leqslant r_0\Sigma_t \leqslant 2.0$. For the rod sizes under consideration, hand calculations using this method are not feasible for enrichments >20%. A single calculation was made for the 0.953-cm-diam rod at 20% enrichment, resulting in a value of $\phi(0)/\phi(r_0) = 0.36$, as opposed to the measured value 0.57 given in Ref. 1.

An alternate method amenable to hand calculation of both the average and the centerline flux in rods has been derived by Bonalumi.[5] To determine the flux within the rod, Bonalumi takes the flux due to a unit cylindrical shell source as the sum of an asymptotic and transient component and then superimposes the contributions due to all such elementary cylindrical shells occupying the space outside the rod.

His result for the flux in a rod of radius r_0 is

$$\phi(r) = I_0(kr) + \lambda T(r_0,r) \ , \tag{4}$$

where k is the positive root of the equation

$$\frac{k}{\Sigma_t} = \tanh \frac{k}{\Sigma_s} \ , \tag{5}$$

$$\lambda = \frac{Dk}{\beta K_1(kr_0)} \ , \tag{6}$$

$$D = \frac{\Sigma_a}{k^2} \ , \tag{7}$$

and β is a coefficient <1, defined by

$$\beta = \frac{2\Sigma_a}{\Sigma_s} \frac{\Sigma_t^2 - k^2}{k^2 - \Sigma_t\Sigma_a} \ . \tag{8}$$

Both k and β are tabulated as functions of Σ_s/Σ_t in Ref. 4. The function T in Eq. (4) is a complicated integral involving products of Bessel functions that can be calculated in finite terms only for $r = 0$ and $r = r_0$. Closed form solutions are given in Ref. 5 for these two cases and for the average value. Results of calculations using Bonalumi's method are shown in Table I. The values calculated using Bonalumi's method are in good agreement with both the results of blackness theory and with the single calculation made using the method of successive generations.

Based on the calculations presented here, it appears that the measured flux ratios reported in Ref. 1 are too high for enrichments >5%, with the error increasing with increasing enrichment. One possible explanation for the anomalous results of Gibson and Anno is that there was a substantial epithermal component to their indium activations, which they failed to take into account. In any event, it has been demonstrated[5] that the flux shape in solid cylindrical rods shows large deviations from the diffusion theory shape even for natural uranium rods. It is not possible to determine a unique value of k_{eff} that will normalize diffusion theory to measured results for use in engineering estimates. Attempts to do so will show that the k_{eff} depends not only on enrichment but on rod diameter and on the measured quantity chosen for normalization. Bonalumi's method[5] is recommended for engineering estimates of the thermal-neutron flux shape in a rod.

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REPLY TO "COMMENTS ON 'THERMAL-NEUTRON FLUX DEPRESSION IN CYLINDRICAL UO_2 FUEL RODS' "

To quote a reviewer of our Note,[1] "the flux depression problem is a difficult one analytically as well as experimentally." We are quite aware of the apparently anomalous experimental results at large enrichment mockups. Indeed, on *theoretical* grounds, the comments[2] on our results appear plausible. In our experiments, we experienced some difficulty in aligning and accurately locating the central axial indium wire. To minimize the possible effect of this problem, every experiment was performed twice and the larger measured flux depression was accepted. The "smoothness" of the curve drawn to the experimental data is evidence of small scatter in the data.

As pointed out in our Note, no correction was made for the perturbation caused by the central 0.51-mm (20-mil)-diam indium wire. This effect is certainly not completely negligible (especially at high-enrichment mockups) considering the diameter of the fuel rods (0.953 cm in most cases). Because of the blackness of the absorber at high-enrichment mockups, the presence of the indium wire would reduce the amount of flux depression from the actual situation, a trend that is in agreement with Sullivan's comments[2] on this Note.

As suggested by Sullivan, probably the most serious difficulty encountered in the experiments is the matter of accounting for epithermal-neutron activation of the indium wires. We failed to comment in the original Note that the measured cadmium ratio for indium (corrected for cadmium cover thickness effects) at the site of the experiment is 7.7.